What did We Learn from the Four Quartic Functions We Graphed Yesterday?

- We can identify all of our x-intercepts from the factored version of the function
- If a factor is twice repeated (ie. \((x-2)^2\) or \((x+3)^2\) ), then the intercept at that value will be a “touch” point
- If a factor is tripled (ie. \((x-2)^3\) or \((x+1)^3\) ), then there will be a point of inflection at the x-intercept
- A regular factor (ie. \((x+2)\) or \((x-4)\) ) will indicate that the function just crosses through the x-axis at that point
- We can determine the end behaviours from the factored version of the graph by determining the degree and looking to see if lead factor is positive or negative
- We can determine the y-intercept from factored form by setting all x’s equal to zero

Examples:

1) \(f(x) = (x - 2)(x + 3)(x - 4)\) degree is 3 (positive lead) , x-intercepts @ x=2, x=-3, x=4 , y-intercept @ y=24

2) \(f(x) = - (x)(x + 3)(x - 4)\) degree is 3 (negative lead) , x-intercepts @ x=0, x=-3, x=4 , y-intercept @ y=0

3) \(f(x) = (x - 1)^2(x + 3)(x - 4)\) degree is 4 (positive lead) , x-intercepts @ x=1, x=-3, x=4 , y-intercept @ y=-12
   ** with a touch point at x=1**

4) \(f(x) = -(x - 2)^3(x + 1)^2(x - 5)\) degree is 6 (neg lead) , x-intercepts @ x=2, x=-1, x=5 , y-intercept @ y=-40
   ** with a touch point at x=-1 and a point of inflection at x=2**

What Role do Factors Play?

Now that we have established a deeper understanding of the role that dominant terms, lead coefficients and factors have on the look of a graph, we are ready to “role” :

Each of the functions are already expressed in **factored form**.

Sketch the graph of each of the following polynomial functions. Your sketch should include proper end behaviours, correct x-intercepts (remember to consider what happens with repeated factors!) and a correct y-intercept.

a) \(f(x) = (x - 4)(x + 3)\)

b) \(f(x) = -(x - 1)(x + 4)(x - \frac{1}{2})\)

c) \(f(x) = (2x - 1)(x + 1)^2\)

d) \(f(x) = 2x(x - 2)^2\)

e) \(f(x) = -(2x - 3)^2(x + 2)^2\)

f) \(f(x) = x(x - 2)(x + 1)(2x + 3)\)

g) \(f(x) = x^3(x - 4)\)

h) \(f(x) = -(x + 3)^2(x - 3)^3\)

i) \(f(x) = x(x + 2)(x - 1)(x - 3)(x + 4)\)

*** solutions to the graphs are on the website