Trigonometric Identities

Trig Identities Worksheet: 1-6 all, 9, 13, 15, 19

I. Reciprocal Identities

A) $\sin\theta = \dfrac{1}{\csc\theta}$ D) $\csc\theta = \dfrac{1}{\sin\theta}$

B) $\cos\theta = \dfrac{1}{\sec\theta}$ E) $\sec\theta = \dfrac{1}{\cos\theta}$

C) $\tan\theta = \dfrac{1}{\cot\theta}$ F) $\cot\theta = \dfrac{1}{\tan\theta}$

II. Quotient Identities

A) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

B) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

III. Pythagorean Identities

A) $\sin^2\theta + \cos^2\theta = 1$

~~B) $\tan^2\theta + 1 = \sec^2\theta$~~

~~C) $\cot^2\theta + 1 = \csc^2\theta$~~

Simplify each expression.

1. $\csc^2\theta - 1$

$= \left(\frac{1}{\sin^2\theta}\right) - 1$

$= \frac{1 - \sin^2\theta}{\sin^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta}$

2. $(1-\sin x)(1+\sin x)$

$= 1 - \sin^2 x$

$= \cos^2 x$

3. $\sin^2\theta(\csc^2\theta - 1)$

$= \sin^2\theta\left(\frac{1}{\sin^2\theta} - 1\right)$

$= 1 - \sin^2\theta$

$= \cos^2\theta$

4. $\cos\theta\tan\theta\csc\theta$

$= \frac{\cos\theta}{1}\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right)$

$= 1$

5. $\dfrac{\csc\theta}{1+\cot^2\theta}$

$= \dfrac{\frac{1}{\sin\theta}}{1 + \frac{\cos^2\theta}{\sin^2\theta}}$

$= \dfrac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta}}$ $= \sin\theta$

6. $\dfrac{1}{\sin^2\theta} - \dfrac{1}{\tan^2\theta}$

$= \dfrac{1}{\sin^2\theta} - \dfrac{1}{\frac{\sin^2\theta}{\cos^2\theta}}$

$= \dfrac{1}{\sin^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta}$

$= \dfrac{1 - \cos^2\theta}{\sin^2\theta}$

$= \dfrac{\sin^2\theta}{\sin^2\theta}$

$= 1$

7. $\sin^2 x + \sin^2 x\cot^2 x$

$= \sin^2 x(1 + \cot^2 x)$

$= \sin^2 x\left(1 + \frac{\cos^2 x}{\sin^2 x}\right)$

$= \sin^2 x + \cos^2 x$

$= 1$

6-3 Trig Identities p. 480: 1-29 odd, omit 3

I. Guidelines for Verifying Trig Identities
   A. Begin with the most complicated side
   B. Work on one side only
   C. Rewrite sums or differences of quotients as one single quotient
   D. Rewrite in terms of sine and cosine only
   E. Factor (GCF or Difference of Squares)
   F. Multiply (Foil or Distributive Property)

Verify each identity.

1. $\csc\theta\sin\theta - \sin^2\theta = \cos^2\theta$

$LS = \frac{1}{\sin\theta}(\sin\theta) - \sin^2\theta$ $RS = \cos^2\theta$

$= 1 - \sin^2\theta$

$= \cos^2\theta$

2. $\sin\theta(\cot\theta + \tan\theta) = \sec\theta$

$RS = \frac{1}{\cos\theta}$

$LS = \sin\theta\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)$

$= \cos\theta + \frac{\sin^2\theta}{\cos\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta} = \frac{1}{\cos\theta}$

3. $(\csc\theta + \cot\theta)(\csc\theta - \cot\theta) = 1$

$RS = 1$

$= \csc^2\theta - \cot^2\theta$

$= \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}$

$= \frac{1-\cos^2\theta}{\sin^2\theta} = \frac{\sin^2\theta}{\sin^2\theta} = 1$

4. $\csc^4\theta - \csc^2\theta = \cot^4\theta + \cot^2\theta$

$LS = \csc^2\theta(\csc^2\theta - 1)$

$= \csc^2\theta\left(\frac{1}{\sin^2\theta} - \frac{\sin^2\theta}{\sin^2\theta}\right)$

$= \frac{1}{\sin^2\theta}\left(\frac{\cos^2\theta}{\sin^2\theta}\right)$

$= \frac{\cos^2\theta}{\sin^4\theta}$

$RS = \frac{\cos^4\theta}{\sin^4\theta} + \frac{\cos^2\theta}{\sin^2\theta}$

$= \frac{\cos^4\theta}{\sin^4\theta} + \frac{\cos^2\theta\sin^2}{\sin^4\theta}$

$= \frac{\cos^2\theta(\cos^2\theta + \sin^2\theta)}{\sin^4\theta}$

$= \frac{\cos^2\theta}{\sin^4\theta}$

5. $\csc x + \cot x = \frac{\sin x}{1-\cos x}$

$LS = \frac{1}{\sin x} + \frac{\cos x}{\sin x}$

$= \frac{1+\cos x}{\sin x}$

$RS = \frac{\sin x}{1-\cos x} \cdot \frac{\sin x}{\sin x}$

$= \frac{1-\cos^2 x}{\sin x(1-\cos x)}$

$= \frac{(1+\cos x)\cancel{(1-\cos x)}}{\sin x\,\cancel{(1-\cos x)}}$

$= \frac{1+\cos x}{\sin x}$

6. $8\csc^2\theta - 3\cot^2\theta = 3 + 5\csc^2\theta$

$LS = \frac{8}{\sin^2\theta} - 3\left(\frac{\cos^2\theta}{\sin^2\theta}\right)$

$= \frac{8 - 3\cos^2\theta}{\sin^2\theta}$

$= \frac{8 - 3(1-\sin^2\theta)}{\sin^2\theta}$

$= \frac{8 - 3 + 3\sin^2\theta}{\sin^2\theta}$

$= \frac{5 + 3\sin^2\theta}{\sin^2\theta}$

$RS = 3 + \frac{5}{\sin^2\theta}$

$= \frac{3\sin^2\theta + 5}{\sin^2\theta}$

7. $\frac{1}{1-\sec\theta} + \frac{1}{1+\sec\theta} = -2\cot^2\theta$

$LS = \frac{1}{1 - \frac{1}{\cos\theta}} + \frac{1}{1 + \frac{1}{\cos\theta}}$

$= \frac{1}{\frac{\cos\theta - 1}{\cos\theta}} + \frac{1}{\frac{\cos\theta + 1}{\cos\theta}}$

$= \frac{\cos\theta}{(\cos\theta - 1)} + \frac{\cos\theta}{(\cos\theta + 1)}$

$\frac{\cos\theta(\cos\theta+1) + \cos\theta(\cos\theta-1)}{\cos^2\theta - 1}$

$= \frac{\cos^2\theta + \cos\theta + \cos^2\theta - \cos\theta}{-\sin^2\theta} = \frac{-2\cos^2\theta}{\sin^2\theta}$

$RS = -2\cot^2\theta$

$= -2\frac{\cos^2\theta}{\sin^2\theta}$