

Trig Identities DAY 2

Prove the following identities:

$$1. \frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} = \frac{2}{\sin^2\theta}$$

$$\begin{aligned}
 \text{LS} &= \frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} \\
 &= \frac{1+\cos\theta}{(1-\cos\theta)(1+\cos\theta)} + \frac{1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)} \\
 &= \frac{2}{(1-\cos\theta)(1+\cos\theta)} \\
 &= \frac{2}{1+\cos\theta - \cos\theta - \cos^2\theta} \\
 &= \frac{2}{1-\cos^2\theta}
 \end{aligned}$$

$$\begin{aligned}
 \text{RS} &= \frac{2}{\sin^2\theta} \\
 &= \frac{2}{1-\cos^2\theta}
 \end{aligned}$$

$$2. (\sin\theta + \cos\theta)(\sin\theta - \cos\theta) = 1 - 2\cos^2\theta$$

$$\begin{aligned}
 \text{LS} &= (\sin\theta + \cos\theta)(\sin\theta - \cos\theta) \\
 &= \sin^2\theta - \sin\theta\cos\theta + \sin\theta\cos\theta - \cos^2\theta \\
 &= \sin^2\theta - \cos^2\theta \\
 &= 1 - \cos^2\theta - \cos^2\theta \\
 &= 1 - 2\cos^2\theta
 \end{aligned}$$

$$\text{RS} = 1 - 2\cos^2\theta$$

$$3. \sin^2\theta - \sin^4\theta = \cos^2\theta - \cos^4\theta$$

$$\begin{aligned} LS &= \sin^2\theta - \sin^4\theta \\ &= \sin^2\theta - (\sin^2\theta)(\sin^2\theta) \\ &= 1 - \cos^2\theta - (1 - \cos^2\theta)(1 - \cos^2\theta) \\ &= 1 - \cos^2\theta - (1 - \cos^2\theta - \cos^2\theta + \cos^4\theta) \\ &= 1 - \cos^2\theta - (1 - 2\cos^2\theta + \cos^4\theta) \\ &= 1 - \cos^2\theta - 1 + 2\cos^2\theta - \cos^4\theta \\ &= \cos^2\theta - \cos^4\theta \end{aligned}$$

$$\begin{aligned} LS &= \sin^2\theta - \sin^4\theta \\ &= \sin^2\theta(1 - \sin^2\theta) \\ &= \sin^2\theta \cos^2\theta \end{aligned}$$

$$\begin{aligned} RS &= \cos^2\theta - \cos^4\theta \\ &= \cos^2\theta(1 - \cos^2\theta) \\ &= \cos^2\theta \sin^2\theta \end{aligned}$$

Text pg 310
5, 8