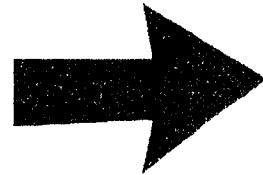


TRIG IDENTITIES

In mathematics, trigonometric identities are equalities that involve trigonometric functions and are true for every single value of the occurring variables.

We have "Basic Identities" that we know to be true....so we use these identities to prove more complicated equalities!



BASIC IDENTITIES

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$
$$\cos^2 \theta = 1 - \sin^2 \theta$$
$$\sin^2 \theta = 1 - \cos^2 \theta$$

Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

Quotient Identities

$$\frac{\sin \theta}{\cos \theta} = \tan \theta \rightarrow \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta$$

$$\frac{\cos \theta}{\sin \theta} = \cot \theta \rightarrow \frac{\cos^2 \theta}{\sin^2 \theta} = \cot^2 \theta$$

Prove the following identities:

$$1. \frac{1 - \cos^2 \theta}{\cos^2 \theta} = \tan^2 \theta$$

$$\begin{aligned} LS &= \frac{1 - \cos^2 \theta}{\cos^2 \theta} & RS &= \tan^2 \theta \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} & &= \frac{\sin^2 \theta}{\cos^2 \theta} \end{aligned}$$

$\because LS = RS$
 \therefore Identity
is proven

$$2. \frac{1}{1 - \sin^2 \theta} = 1 + \tan^2 \theta$$

$$\begin{aligned} LS &= \frac{1}{1 - \sin^2 \theta} & RS &= 1 + \tan^2 \theta \\ &= \frac{1}{\cos^2 \theta} & &= 1 + \frac{\sin^2 \theta}{\cos^2 \theta} \\ & & &= \frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} \\ & & &= \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} \\ & & &= \frac{1}{\cos^2 \theta} \end{aligned}$$

$$3. \frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$$

$$LS = \frac{1-\sin\theta}{\cos\theta} \quad RS: \frac{\cos\theta}{1+\sin\theta}$$

$$\begin{aligned} &= \frac{1-\sin\theta}{\cos\theta} \times \frac{1+\sin\theta}{1+\sin\theta} \\ &= \frac{1+\sin\theta - \sin\theta - \sin^2\theta}{\cos\theta(1+\sin\theta)} \\ &= \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)} \\ &= \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)} \end{aligned}$$

$$\begin{aligned} &= \frac{(\cos\theta)(\cos\theta)}{(\cos\theta)(1+\sin\theta)} \\ &= \frac{\cos\theta}{1+\sin\theta} \end{aligned}$$

$\therefore LS = RS$
 \therefore Identity
 is proven

Prove the following identities:

$$1. \frac{\sin\theta}{\tan\theta} = \cos\theta$$

$$2. \sin\theta \cos\theta \tan\theta = 1 - \cos^2\theta$$

$$3. 2\sin^2\theta - 1 = \sin^2\theta - \cos^2\theta$$

$$4. \frac{1}{\cos\theta} - \cos\theta = \sin\theta \tan\theta$$

$$5. \sin\theta + \tan\theta = \tan\theta(1 + \cos\theta)$$