# FACTOR and REMAINDER Theorems

THINK & COMM

## Version #1

1.(a) $f(x) = 2x^3 + x^2 - 27x - 36$

$$f(-2) = 2(-2)^3 + (-2)^2 - 27(-2) - 36$$
$$= -16 + 4 + 54 - 36$$

$$f(-3) = 2(-3)^3 + (-3)^2 - 27(-3) - 36$$
$$= -54 + 9 + 81 - 36$$
$$= 0$$

$\therefore$ $(x+3)$ is a factor

| | | | | |
|---|---|---|---|---|
| $-3$ | 2 | 1 | $-27$ | $-36$ |
| | $\downarrow$ | $-6$ | 15 | 36 |
| | 2 | $-5$ | $-12$ | 0 R |

$$f(x) = (x+3)(2x^2 - 5x - 12)$$
$$= \underset{l}{(x+3)}\underset{w}{(2x+3)}\underset{h}{(x-4)}$$

$2^{3}$ $4^{2}$ $25.5$

$1^{2}$ $-4^{3}$

(b) SA

| $l \times w$ | $l \times h$ | $w \times h$ |
|---|---|---|
| $(x+3)(2x+3)$ | $(x+3)(x-4)$ | $(2x+3)(x-4)$ |
| $= 2x^2 + 9x + 9$ | $= x^2 - x - 12$ | $= 2x^2 - 5x - 12$ |

$$SA = 2(2x^2 + 9x + 9) + 2(x^2 - x - 12) + 2(2x^2 - 5x - 12)$$
$$= 4x^2 + 18x + 18 + 2x^2 - 2x - 24 + 4x^2 - 10x - 24$$
$$= 10x^2 + 6x - 30$$

If $x = 5.5$ $\quad SA = 10(5.5)^2 + 6(5.5) - 30$
$$= 302.5 + 33 - 30$$
$$= 305.5 \text{ cm}^2$$

2. $f(x) = 3x^2 + 10x - 3$

$f(-K) = 5$

$\therefore \quad 3(-K)^2 + 10(-K) - 3 = 5$

$3 \quad 2$
$1 \quad -4$

$3K^2 - 10K - 8 = 0$

$(3K+2)(K-4) = 0$

$K = -\frac{2}{3} \qquad K = 4$

3. (a) $f(x) = ax^3 - x^2 + 2x + b$

$f(1) = 8$

$f(2) = 28$

$a(1)^3 - (1)^2 + 2(1) + b = 8$

$a - 1 + 2 + b = 8$

① $a + b = 7$

$a(2)^3 - (2)^2 + 2(2) + b = 28$

$8a - 4 + 4 + b = 28$

② $8a + b = 28$

② $8a + b = 28$

① $a + b = 7$

Subtract $7a = 21$

$\boxed{a = 3} \quad \therefore \boxed{b = 4}$

(b) $f(x) = 3x^3 - x^2 + 2x + 4 \quad \div (x^2 - 2)$

$$
\begin{array}{r l}
 & 3x - 1 \\
x^2 + 0x - 2 \,\big) & 3x^3 - x^2 + 2x + 4 \\
 & \underline{3x^3 + 0x^2 - 6x} \quad \downarrow \\
 & -x^2 + 8x + 4 \\
 & \underline{-x^2 + 0x + 2} \\
 & 8x + 2
\end{array}
$$

quotient: $(3x - 1)$
remainder: $(8x + 2)$

(c) Division Statement

$$3x^3 - x^2 + 2x + 4 = (x^2-2)(3x-1) + (8x+2)$$

$$\begin{aligned} RS &= (x^2-2)(3x-1) + (8x+2) \\ &= 3x^3 - x^2 - 6x + 2 + 8x + 2 \\ &= 3x^3 - x^2 + 2x + 4 \end{aligned}$$

$\therefore$ division statement is true

4. $f(x) = 6x^3 - 5x^2 + kx - 18 \quad \div (2x-3)$

FIND "K"

$$f\left(\tfrac{3}{2}\right) = -3$$

$$\therefore 6\left(\tfrac{3}{2}\right)^3 - 5\left(\tfrac{3}{2}\right)^2 + k\left(\tfrac{3}{2}\right) - 18 = -3$$

$$6\left(\tfrac{27}{8}\right) - 5\left(\tfrac{9}{4}\right) + k\left(\tfrac{3}{2}\right) = 15$$

$$\frac{81}{4} - \frac{45}{4} + \frac{6k}{4} = 15$$

$$9 + \frac{6k}{4} = 15$$

$$\frac{6k}{4} = 6$$

$$6k = 24$$

$$\boxed{k = 4}$$

FIND the Quotient

$\therefore f(x) = 6x^3 - 5x^2 + 4x - 18 \quad \div (2x-3)$

$$\begin{array}{c|cccc} \frac{3}{2} & 6 & -5 & 4 & -18 \\ & \downarrow & 9 & 6 & 15 \\ \hline & 6 & 4 & 10 & -3 \end{array} \; R$$

$\div 2$

$\boxed{3x^2 + 2x + 5}$ QUOTIENT