Piecewise Functions

In Summary

Key Ideas

- Some functions are represented by two or more "pieces." These functions are called piecewise functions.
- Each piece of a piecewise function is defined for a specific interval in the domain of the function.

Need to Know

- To graph a piecewise function, graph each piece of the function over the given interval.
- A piecewise function can be either continuous or not. If all the pieces of the function join together at the endpoints of the given intervals, then the function is continuous. Otherwise, it is discontinuous at these values of the domain.

1. Graph each piecewise function.

a) $f(x) = \begin{cases} 2, \text{if } x < 1 \\ 3x, \text{if } x \geq 1 \end{cases}$

b) $f(x) = \begin{cases} -2x, \text{if } x < 0 \\ x + 4, \text{if } x \geq 0 \end{cases}$

c) $f(x) = \begin{cases} |x|, \text{if } x \leq -2 \\ -x^2, \text{if } x > -2 \end{cases}$

d) $f(x) = \begin{cases} |x + 2|, \text{if } x \leq -1 \\ -x^2 + 2, \text{if } x > -1 \end{cases}$

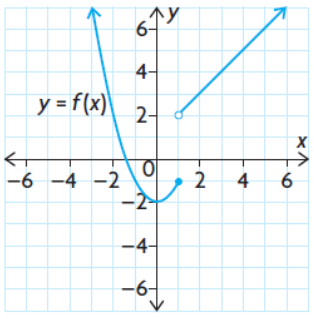
e) $f(x) = \begin{cases} \sqrt{x}, \text{if } x < 4 \\ 2^x, \text{if } x \geq 4 \end{cases}$

f) $f(x) = \begin{cases} \frac{1}{x}, \text{if } x < 1 \\ -x, \text{if } x \geq 1 \end{cases}$

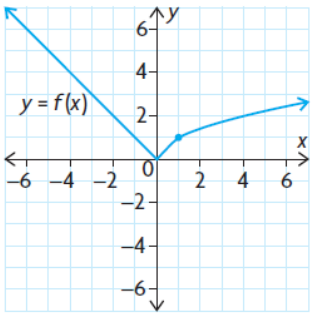
2. State whether each function in question 1 is continuous or not. If not, state where it is discontinuous.

3. Write the algebraic representation of each piecewise function, using function notation.

a)



b)



4. State the domain of each piecewise function in question 3, and comment on the continuity of the function.

5. K Graph the following piecewise functions. Determine whether each function is continuous or not, and state the domain and range of the function.

a) $f(x) = \begin{cases} 2, \text{ if } x < -1 \\ 3, \text{ if } x \geq -1 \end{cases}$

b) $f(x) = \begin{cases} -x, \text{ if } x \leq 0 \\ x, \text{ if } x > 0 \end{cases}$

c) $f(x) = \begin{cases} x^2 + 1, \text{ if } x < 2 \\ 2x + 1, \text{ if } x \geq 2 \end{cases}$

d) $f(x) = \begin{cases} 1, \text{ if } x < -1 \\ x + 2, \text{ if } -1 \leq x \leq 3 \\ 5, \text{ if } x > 3 \end{cases}$

8. T Find the value of k that makes the following function continuous. Graph the function.

$$f(x) = \begin{cases} x^2 - k, \text{ if } x < -1 \\ 2x - 1, \text{ if } x \geq -1 \end{cases}$$

9. The fish population, in thousands, in a lake at any time, x, in years is modelled by the following function:

$$f(x) = \begin{cases} 2^x, \text{ if } 0 \leq x \leq 6 \\ 4x + 8, \text{ if } x > 6 \end{cases}$$

This function describes a sudden change in the population at time $x = 6$, due to a chemical spill.

a) Sketch the graph of the piecewise function.
b) Describe the continuity of the function.
c) How many fish were killed by the chemical spill?
d) At what time did the population recover to the level it was before the chemical spill?
e) Describe other events relating to fish populations in a lake that might result in piecewise functions.

14. Explain why there is no value of k that will make the following function continuous.

$$f(x) = \begin{cases} 5x, \text{ if } x < -1 \\ x + k, \text{ if } -1 \leq x \leq 3 \\ 2x^2, \text{ if } x > 3 \end{cases}$$