Summary - Graphs of Trigonometric Functions

General Equations
$y = A\sin k(\theta + p) + q$
$y = A\cos k(\theta + p) + q$

Vertical Stretch (Amplitude)	A
Horizontal Stretch (Period Length)	$PL = \frac{2\pi}{k}$
Vertical Shift	+q is up -q is down
Horizontal Shift (Phase Shift)	+p is left -p is right
Reflection over x-axis	-A

Examples

State the vertical shift (the equation of the central axis), the phase shift, the amplitude and the period length for each of the following:

1) $y = -2\sin\left(\theta - \frac{\pi}{2}\right) + 4$ $k=1$

mapping $(\theta, y) \longrightarrow \left(\theta + \frac{\pi}{2}, -2y+4\right)$

θ	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

$\longrightarrow$

θ	y
$\pi/2$	4
π	2
$3\pi/2$	4
2π	6
$\frac{5\pi}{2}$	4

Vertical Shift (equation central axis) $\boxed{y=4}$

phase Shift $\boxed{\text{right } \pi/2}$

Amp $\boxed{2}$

Period Length

$PL = \frac{2\pi}{k} = \frac{2\pi}{1} = \boxed{2\pi}$

2) $y = \frac{1}{3}\sin\left(3\theta + \frac{\pi}{4}\right) - 7$

*Factor First

$y = \frac{1}{3}\sin\left(3\left(\theta + \frac{\pi}{12}\right)\right) - 7$

$(\theta, y) \to \left(\frac{1}{3}\theta - \frac{\pi}{12}, \frac{1}{3}y - 7\right)$

$\frac{\pi}{4} \div 3$

$\frac{\pi}{4} \times \frac{1}{3} = \pi/12$

Vs $\boxed{y=-7}$

PS $\boxed{\text{left } \pi/12}$

Amp $\boxed{\frac{1}{3}}$

PL $\boxed{\frac{2\pi}{3}}$

Understanding Period Length, Phase Shifts and Amplitude

Fill in the following table:

Function	Vertical Shift ~~Central axis~~	Phase Shift	Amplitude	Period Length
$y = 3\sin\theta$	$y=0$	none	3	2π
$y = 2\cos\theta - 3$	$y=-3$	none	2	2π
$y = 2\cos\left(\theta - \frac{\pi}{3}\right)$	$y=0$	right $\pi/3$	2	2π
$y = \sin\left(\theta + \frac{\pi}{4}\right) - 1$	$y=-1$	left $\pi/4$	1	2π
$y = -5\cos 3\theta - 2$	$y=-2$	none	5	$2\pi/3$
$y = 4\sin\left(\theta - \frac{2\pi}{3}\right) - 1$	$y=-1$	right $2\pi/3$	4	2π
$y = \sin(2\theta - \pi) + 3$ $\hookrightarrow y = \sin\left(2\left(\theta - \frac{\pi}{2}\right)\right) + 3$	$y=3$	right $\pi/2$	1	π
$y = -2\cos\left(4\theta - \frac{\pi}{2}\right) - 1$ $\hookrightarrow y = -2\cos\left(4\left(\theta - \frac{\pi}{8}\right)\right) - 1$	$y=-1$	right $\pi/8$	2	$\pi/2$