Wed / Thurs

Solving Polynomial Equations

Solve each of the following (find the zeros, find the roots):

Example 1

$x^3 - 5x^2 - 2x + 24 = 0$

"Factor Theorem"

$f(-2) = 0$

$\therefore (x+2)$ factor

$$\begin{array}{r|rrrr} -2 & 1 & -5 & -2 & 24 \\ & & -2 & 14 & -24 \\ \hline & 1 & -7 & 12 & 0 \end{array}$$

$\therefore (x+2)(x^2-7x+12) =$

$(x+2)(x-4)(x-3) =$

$\therefore x = -2, 4, 3$

Example 2

$x^3 - x^2 - 9x + 9 = 0$

"Grouping"

$x^2(x-1) - 9(x-1) = 0$

$(x^2-9)(x-1) = 0$

$(x+3)(x-3)(x-1) = 0$

$\therefore x = -3, 3, 1$

Example 3

$4x^4 + 6x^3 - 6x^2 - 4x = 0$

"Common First"

$2x(2x^3 + 3x^2 - 3x - 2) = 0$

$f(1) = 0$

$\therefore (x-1)$ factor

$$\begin{array}{r|rrrr} 1 & 2 & 3 & -3 & -2 \\ & & 2 & 5 & 2 \\ \hline & 2 & 5 & 2 & 0 \end{array}$$

$\therefore 2x(x-1)(2x^2+5x+2) =$

$2x(x-1)(2x+1)(x+2) = 0$

$\therefore x = 0, 1, -\frac{1}{2}, -2$

Example 4

$x^4 - 1 = 0$

"diff of squares"

$(x^2-1)(x^2+1) = 0$

$(x+1)(x-1)(x^2+1) = 0$

$x^2 = -1$

$x = \pm\sqrt{-1}$

$x = \pm i$

$\therefore x = -1, 1, \pm i$

is not a "real" root on the graph

Example 5

$(x^2 - 5x - 5)(x^2 - 5x + 3) = 9$

"expand" get in right form

$x^4 - 10x^3 + 23x^2 + 10x - 24 = 0$

$f(1) = 0 \quad \therefore (x-1)$ is a factor

$$\begin{array}{r|rrrrr} 1 & 1 & -10 & 23 & 10 & -24 \\ & & 1 & -9 & 14 & 24 \\ \hline & 1 & -9 & 14 & 24 & 0 \end{array}$$

$x^3 - 9x^2 + 14x + 24$

$(x-6)(x+1)(x-4)(x-1) = 0$

Example 6

over

Example 6

-15
$9h)$

$$(x+1)(x+5)(x+3) = -3$$

$$(x+1)(x^2+8x+15) = -3$$

$$x^3+8x^2+15x+x^2+8x+15+3=0$$

$$x^3+9x^2+23x+18=0$$

$$f(-2) = -8+9(4)\overset{-46}{\cancel{}}+18$$
$$=0$$

$\therefore (x+2)$ is factor

$$\therefore -2 \left|\begin{array}{cccc} 1 & 9 & 23 & 18 \\ & -2 & -14 & -18 \\ \hline 1 & 7 & 9 & 0 \end{array}\right.$$

$$\therefore (x+2)(x^2+7x+9)=0$$

cannot factor!

$$x=-2$$

$$\frac{-7\pm\sqrt{49-36}}{2}$$
$$=\frac{-7\pm\sqrt{13}}{2}$$

$\therefore$ roots are

$$x=-2,\ \frac{-7+\sqrt{13}}{2},\ \frac{-7-\sqrt{13}}{2}$$