Relations and Functions

Relations

A relation is ____.

Examples:

Functions

A function is a relation, ____.

By definition, *A* is not a function, whereas *B* is a function.

We can represent *A* and *B* using ***mapping diagrams***.

Representing Relations and Functions Graphically

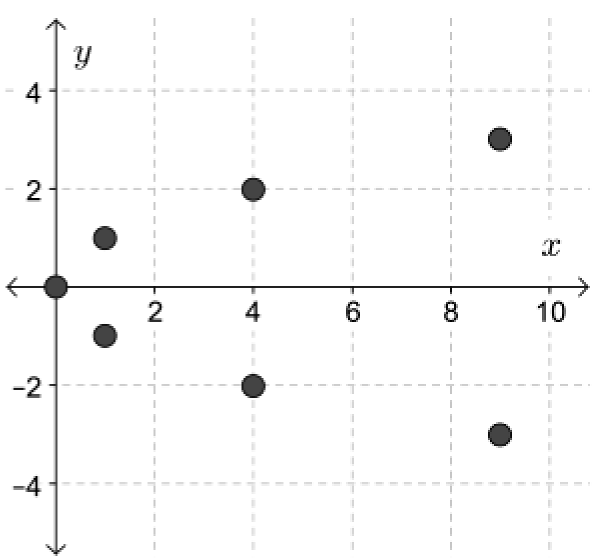
We have represented A and B using set notation and mapping diagrams.

We can also represent A and B on a graph.

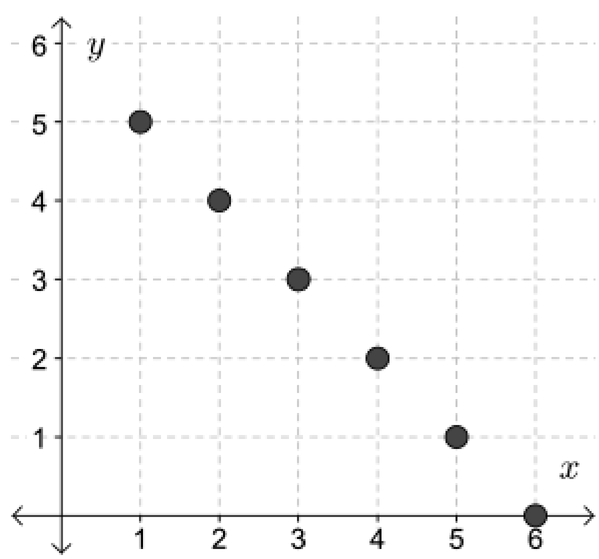
$A=\{(9,-3),(4,-2),(1,-1),(0,0),(1,1),(4,2),(9,3)\}$

$B=\{(1,5),(2,4),(3,3),(4,2),(5,1),(6,0)\}$

Set A



Set B



The Vertical Line Test

If a vertical line can be drawn anywhere on a graph so that it passes through two or more points on a relation, then

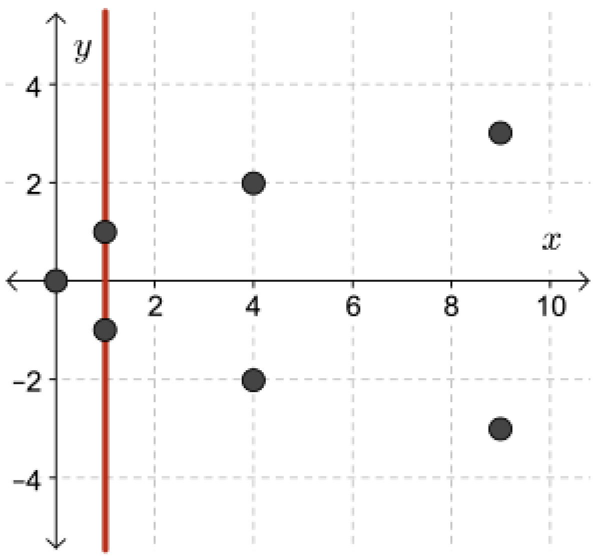
____.

However, if no vertical line can be drawn that passes through more than one point on a relation, then

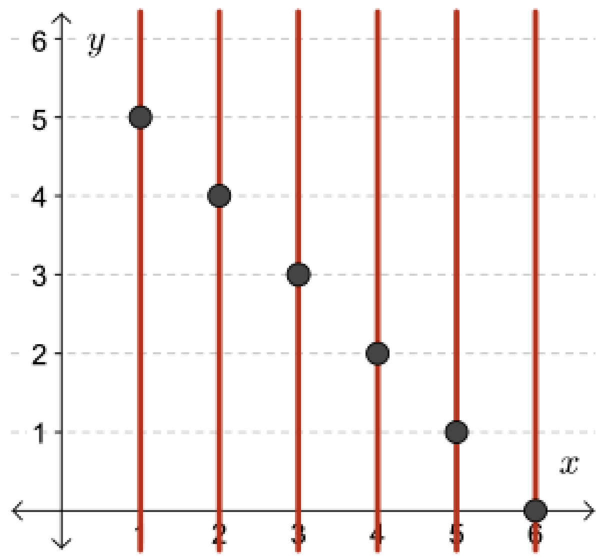
____.

This is called the ________________________________.

Set A



Set B



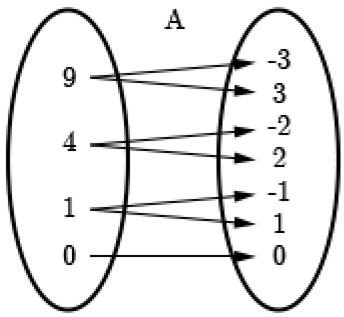
Conclusion:

Set A: A vertical line can be drawn through the two points, $(1,-1)$ and $(1,1)$. Therefore, A is ________________.

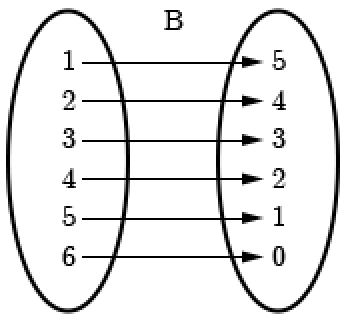
Set B: No vertical line can be drawn through any two points on B. Therefore, B is ________________________.

Domain and Range

$A = \{(9,-3),(4,-2),(1,-1),(0,0),(1,1),(4,2),(9,3)\}$



$B = \{(1,5),(2,4),(3,3),(4,2),(5,1),(6,0)\}$



The set of all possible values of the independent variable, x, is called the ___________________.
The set of all possible values of the dependent variable, y, is called the ____________________.

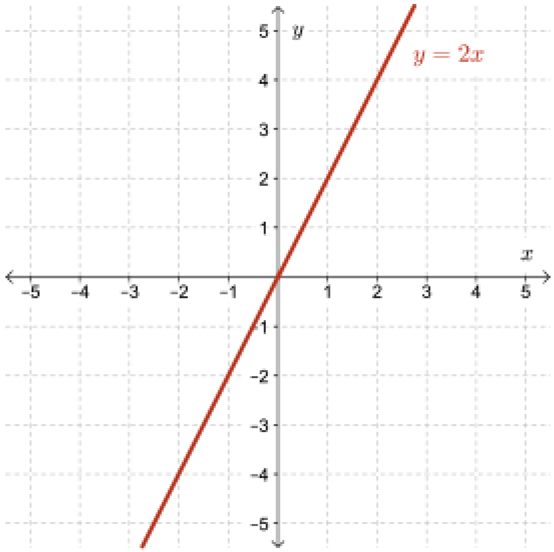
For
$A = \{(9,-3),(4,-2),(1,-1),(0,0),(1,1),(4,2),(9,3)\}$
the domain is $\{0,1,4,9\}$, and
the range is $\{-3,-2,-1,0,1,2,3\}$.

For
$B = \{(1,5),(2,4),(3,3),(4,2),(5,1),(6,0)\}$
the domain is $\{1,2,3,4,5,6\}$, and
the range is $\{0,1,2,3,4,5\}$.

Domain and Range

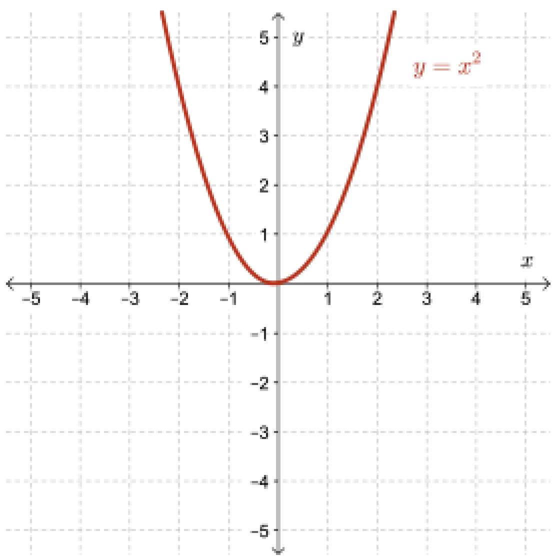
Example 1

State the domain and range of the following. State, with justification, whether or not each relation is a function.



In set notation, the symbol, "|", is a mathematical short form for "such that" and the symbol, "∈", stands for "element of."

Example 2



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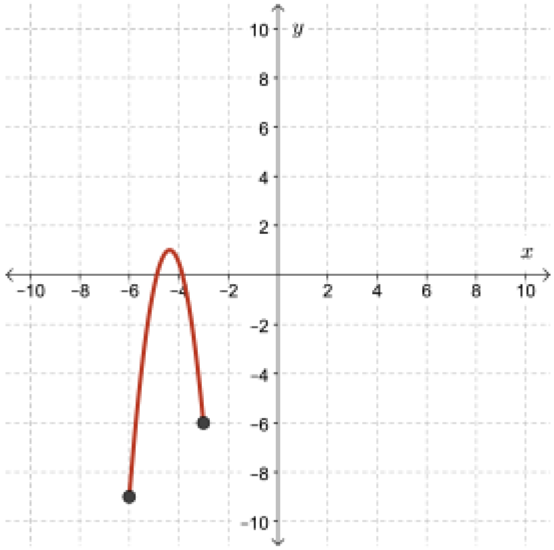
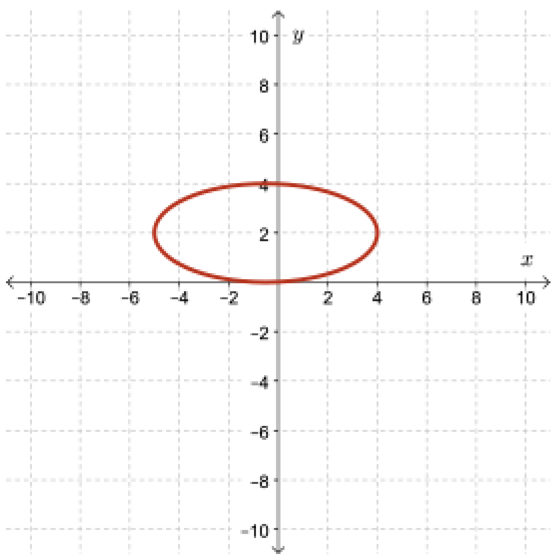
Example 3



Example 4



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Function Notation

Example 1

Let $f(x)=2x+3$ and $g(x)=x^2+4x$.

a. Evaluate

- **i.** $f(-6)$
- **ii.** $g(-3)$

Example 2

Let $f(x)=2x+3$ and $g(x)=x^2+4x$.

b. For what value(s) of x does $f(x)=-8$?

Example 3

Let $f(x)=2x+3$ and $g(x)=x^2+4x$.

c. For what value(s) of x does $f(x)=g(x)$?

Example 4

Let $f(x)=2x+3$ and $g(x)=x^2+4x$.

d. Evaluate $f(g(5))$.

Example 5

Let $f(x)=2x+3$ and $g(x)=x^2+4x$.

e. Simplify $f(g(x))$.

f. Simplify $g(f(x))$

Composite Functions

$$f(x)=2x+3$$
$$g(x)=x^2+4x$$
$$f(g(x))=2x^2+8x+3 \text{ and } g(f(x))=4x^2+20x+21$$

$f(g(x))$ and $g(f(x))$ are examples of ________________________________..

Composition of functions is the process of combining two or more functions where one function is performed first and the result is substituted in place of x into the next function, and so on. When we substitute one function into another function, we create a ____..

$f(g(x))$ is read "f of g of x". It is also written $(f \circ g)(x)$.

The idea of composite functions will be developed in other modules of this course.
We will answer questions like, "When is it possible to compose two functions?", and, "Do two functions exist such that $f(g(x))=g(f(x))$?"

Summary of Functions

The concepts introduced in this module may not have been totally new, but they provide some of the basic building blocks for further study of functions.

- We introduced relations and functions.
- We represented relations and functions using set notation, mapping diagrams, and graphs.
- We introduced the vertical line test.
- We introduced domain and range.
- We introduced function notation.
- We introduced composite functions.