

(2)

Quadratics - Finding Max & Min Values (Applications)

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Worksheet (9)-18

$$(8) h(t) = -5t^2 + 20t + 50$$

$$a) h(t) = -5(t^2 - 4t) + 50$$

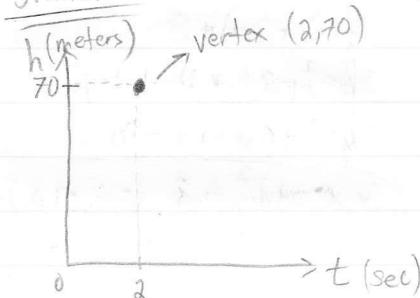
$$h(t) = -5((t^2 - 4t + 4) - 4) + 50$$

$$h(t) = -5(t-2)^2 + 20 + 50$$

$$h(t) = -5(t-2)^2 + 70$$

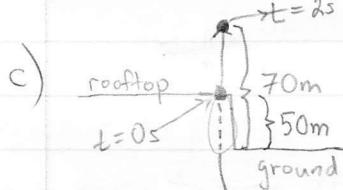
\therefore Max height is 70 m

Sketch!



b) The ball is at its max height at the vertex $(2, 70)$ so at

$$t = 2 \text{ seconds}$$



We know the initial height of the ball is when $t=0$, so $h(0) = -5(0)^2 + 20(0) + 50$
 $h(0) = 50\text{m}$

\therefore The height of the rooftop above the ground is 50m

$$(9) C(x) = 0.28x^2 - 0.7x + 1$$

$$C(x) = 0.28(x^2 - 2.5x) + 1$$

$$C(x) = 0.28((x^2 - 2.5x + 1.5625) - 1.5625) + 1$$

$$C(x) = 0.28(x - 1.25)^2 - 0.4375 + 1$$

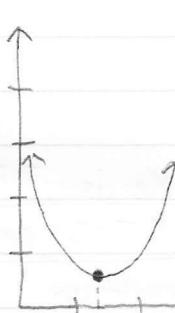
$$C(x) = 0.28(x - 1.25)^2 + 0.5625$$

- Vertex is $(1.25, 0.5625)$

- Parabola opens up so we have a min value at $x = 1.25$

$$y = 0.5625$$

$C(x)$
cost per hour
(in millions)



x items produced per hour
(in thousands)

So then what is the minimum production cost? Since we know $C(x)$ or y is the production cost and our min is at $y = 0.5625$, then the min cost (in millions) is $\$1000000(0.5625) = \$562,500$

⑯ $3x^2 - 6x + 5$ cannot be less than 1

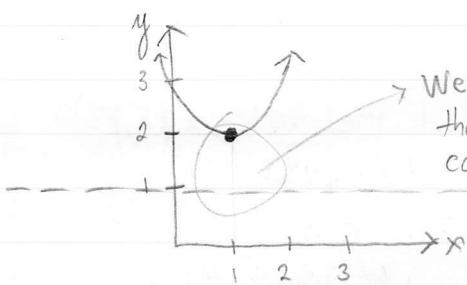
$$3(x^2 - 2x) + 5$$

$$3((x^2 - 2x + 1) - 1) + 5$$

$$3(x^2 - 2x + 1) - 3 + 5$$

$$3(x-1)^2 + 2$$

\therefore Parabola opens up so we have a min at $(1, 2)$



We have a min at $y = 2$, therefore the value of $3x^2 - 6x + 5$ cannot be less than 1!

⑪ $P(x) = -5x^2 + 400x - 2550$ where (x is amount spent in advertising) $P(x)$ is profit of cosmetics company

$$P(x) = -5(x^2 - 80x) - 2550$$

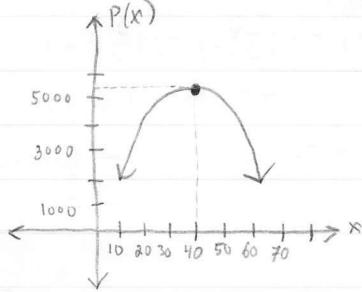
$$P(x) = -5((x^2 - 80x + 1600) - 1600) - 2550$$

$$P(x) = -5(x-40)^2 + 8000 - 2550$$

$$P(x) = -5(x-40)^2 + 5450$$

- Opens down

- Therefore max at $(40, 5450)$



a) The max profit, $P(x)$, the company can make is 5450 (at vertex) in thousands of dollars. $\therefore \$1000(5450) = \$5450000 \leftarrow$

b) The amount spent on advertising that would result in the max profit from part a) would be 40 (ie/our max is at 5450 when $x=40$) in thousands of dollars. $\therefore \$1000(40) = \40000

c) We want to make at least $\$4000000 = P(x)$ and find x (amount spent advertising). $\therefore P(x) = -5(x-40)^2 + 5450$

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Don't forget to divide this by \$1000 so we can use it in the equation!!

$$\$4000000 = -5(x-40)^2 + 5450$$

$$4000 = -5(x-40)^2 + 5450$$

$$4000 - 5450 = -5(x-40)^2$$

$$\frac{-1450}{-5} = (x-40)^2$$

$$290 = (x-40)^2$$

$$\pm\sqrt{290} = x-40$$

$$40 \pm \sqrt{290} = x$$

$$1. x = 40 + \sqrt{290}$$

$$x = 57.029386$$

$$x = \$1000(57.029386)$$

$$x = \$57029$$

$$\& x = 40 - \sqrt{290}$$

$$x = 22.97061$$

$$x = \$1000(22.97061)$$

$$x = \$22971$$

\therefore You must spend between \$22971 & \$57029 to obtain a profit of at least \$4000000.

Maximum Fun Worksheet!

⑩ Two numbers have a sum of 13.

a) $\underline{?} + \underline{?} = 13$

$$x + \underline{?} = 13$$

$$\underline{?} = 13 - x$$

Let the first number be x , what's the second?

The second number is $(13 - x)$

Let this be y

We are looking for the minimum product of the sum of their squares.

$$\therefore y = (x)^2 + (13-x)^2$$

$$y = x^2 + (13-x)(13-x)$$

$$y = x^2 + 169 - 26x + x^2$$

$$y = 2x^2 - 26x + 169$$

$$y = 2(x^2 - 13x) + 169$$

$$y = 2\left(x^2 - 13x + \frac{169}{4}\right) - \frac{169}{4} + 169$$

$$y = 2\left(x - \frac{13}{2}\right)^2 - \frac{169}{2} + \frac{338}{2}$$

$$y = 2\left(x - \frac{13}{2}\right)^2 + \frac{169}{2}$$

$$\text{Min at } \left(\frac{13}{2}, \frac{169}{2}\right)$$

x y Since y is product of sum of squares, the minimum product of sum of squares can be found at the vertex.

$$\therefore \text{At } y = \frac{169}{2} = 84.5$$

b) We know that the first number is $x = \frac{13}{2}$ ①

The second number is $13 - x$

$$= 13 - \frac{13}{2}$$

$$= \frac{13}{2}$$

\therefore The two numbers are $\frac{13}{2} \pm \frac{13}{2}$ or 6.5 & 6.5.

$$\text{11) a) } y = x^2 - 9$$

$$y = (x + 0)^2 - 9$$

Notice that this is already in vertex form, but there was no horizontal shift! Don't be thrown off, always look for that vertex form when trying to find a max/min!

So our parabola opens up and our vertex is $(0, -9)$

\therefore We have a min at -9 when $x=0$

$$\text{b) } y = -4x^2 + 25$$

$$y = -4(x^2) + 25$$

$$y = -4(x+0)^2 + 25$$

Our parabola opens down and our vertex is $(0, 25)$

\therefore We have a max at 25 when $x=0$

$$\text{12) } A = \frac{1}{2}bh \quad \textcircled{1}$$

$$b+h = 13 \quad \textcircled{2}$$

From $\textcircled{2}$, $b+h=13$

$$h = 13 - b$$

Substitute this $(h=13-b)$ into equation $\textcircled{1}$

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}b(13-b)$$

$$A = \frac{1}{2}(13b - b^2)$$

$$A = \frac{13}{2}b - \frac{1}{2}b^2$$

$$A = -\frac{1}{2}b^2 + \frac{13}{2}b$$

(Let's get this quadratic into vertex form so we can find the max!)

$$A = -\frac{1}{2}(b^2 - 13b)$$

$$A = -\frac{1}{2}\left[\left(b^2 - 13b + \frac{169}{4}\right) - \frac{169}{4}\right]$$

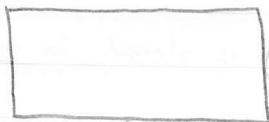
$$A = -\frac{1}{2}\left(b^2 - 13b + \frac{169}{4}\right) + \frac{169}{8}$$

$$A = -\frac{1}{2}\left(b - \frac{13}{2}\right)^2 + \frac{169}{8}$$

\therefore We have a max area of $\frac{169}{8} \text{ cm}^2$ when the base is $\frac{13}{2} \text{ cm}$.

Don't forget the units!

(13)

 $5-2x$ → width!

3x → length!

a) $A_{\text{rectangle}} = (\text{length})(\text{width})$

$$A = (3x)(5-2x)$$

$$A = 15x - 6x^2$$

$$A = -6x^2 + 15x$$

(Let's put this quadratic into vertex form so we can find the max area of the rectangle.)

$$A = -6(x^2 - \frac{15}{6}x)$$

$$A = -6\left[\left(x^2 - \frac{15}{6}x + \frac{225}{144}\right) - \frac{225}{144}\right]$$

$$A = -6\left(x^2 - \frac{15}{6}x + \frac{225}{144}\right) + \frac{1350}{144}$$

$$A = -6\left(x - \frac{15}{12}\right)^2 + \frac{75}{8}$$

Aside: $\left(-\frac{15}{6} \div 2\right)^2$

$$= \left(-\frac{15}{6} \times \frac{1}{2}\right)^2$$

$$= \left(-\frac{15}{12}\right)^2$$

$$= \frac{225}{144}$$

(Our magic number)

- b) \therefore Our max area is $\frac{75}{8}$ (or 9.375 units²) and occurs when $x = \frac{15}{12}$ (or $x = 1.25$ units)

(14) $h = -\frac{1}{2}gt^2 + v_0t + h_0$

The question tells us a few things:

i) $g = 9.8 \text{ m/s}^2$ (acceleration due to gravity)

ii) $v_0 = 34.3 \text{ m/s}$ (initial velocity)

iii) $h_0 = 2.1 \text{ m}$ (initial height)

Let's sub all these values into our equation:

$$h = -\frac{1}{2}gt^2 + v_0t + h_0$$

$$h = -\frac{1}{2}(9.8)t^2 + (34.3)t + 2.1$$

$$h = -4.9t^2 + 34.3t + 2.1$$

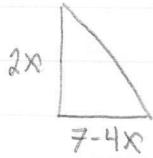
Now we just have to turn this standard form quadratic into a vertex form quadratic to find the max height!

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$$\begin{aligned}
 h &= -4.9(t^2 - 7t) + 2.1 \\
 h &= -4.9[(t^2 - 7t + 12.25) - 12.25] + 2.1 \\
 h &= -4.9(t^2 - 7t + 12.25) + 60.025 + 2.1 \\
 h &= -4.9(t - 3.5)^2 + 62.125 \leftarrow (\text{Vertex Form})
 \end{aligned}$$

a) & b) \therefore The max height is 62.125 m and this occurs at $t = 3.5$ seconds after launch.

(15)



$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(7-4x)(2x)$$

$$A = \frac{1}{2}(14x - 8x^2)$$

$$A = 7x - 4x^2$$

$$A = -4x^2 + 7x \leftarrow (\text{Get into vertex form!})$$

$$A = -4(x^2 - \frac{7}{4}x)$$

$$A = -4\left[\left(x^2 - \frac{7}{4}x + \frac{49}{64}\right) - \frac{49}{64}\right]$$

$$A = -4\left(x^2 - \frac{7}{4}x + \frac{49}{64}\right) + \frac{49}{16}$$

$$A = -4\left(x - \frac{7}{8}\right)^2 + \frac{49}{16} \Rightarrow \text{Vertex is } \left(\frac{7}{8}, \frac{49}{16}\right)$$

a)+b) \therefore Max area of triangle is $\frac{49}{16}$ (or 3.0625 units²) when $x = \frac{7}{8}$ (or 0.875 units)

(16)

$$h = -0.004d^2 + 0.14d + 2$$

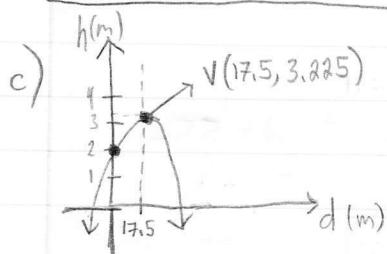
$$a) h = -0.004(d^2 - 35d) + 2$$

$$h = -0.004[(d^2 - 35d + 306.25) - 306.25] + 2$$

$$h = -0.004(d^2 - 35d + 306.25) + 1.225 + 2$$

$$h = -0.004(d - 17.5)^2 + 3.225 \Rightarrow \text{Vertex is } (17.5, 3.225)$$

\therefore Max height is at 3.225 m and b) occurs when the ball has travelled a horizontal distance of $d = 17.5$ m



We know that when the ball is released it has travelled a horizontal distance of 0 m, so to find the initial height, we just need to find the h -intercept (or y -intercept!).

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h-intercept: Set $d = 0$

$$\therefore h = -0.004(0)^2 + 0.14(0) + 2$$

$$h = 2 \text{ m}$$

\therefore The ball is 2m from the ground when the player releases it.

(17) $S = -0.008t^2 + 0.04t$

$$S = -0.008(t^2 - 5t)$$

$$S = -0.008\left[\left(t^2 - 5t + \frac{25}{4}\right) - \frac{25}{4}\right]$$

$$S = -0.008\left(t^2 - 5t + \frac{25}{4}\right) + 0.05$$

$$S = -0.008\left(t - \frac{5}{2}\right)^2 + 0.05$$

$$S = -0.008(t - 2.5)^2 + 0.05$$

Opens down, max at $(2.5, 0.05)$
 $t(h)$ $S(\text{mm}^2/\text{h})$

We know S is the rate of increase in surface area in mm^2/h
so our max rate of increase is at the vertex. We also know
the time taken to reach this maximum is at the vertex!

$$\therefore \text{max of } 0.05 \text{ mm}^2/\text{h} \text{ when } t = 2.5 \text{ h}$$

(18) $h = -0.043d^2 + 2.365d$

a) $h = -0.043(d^2 - 55d)$

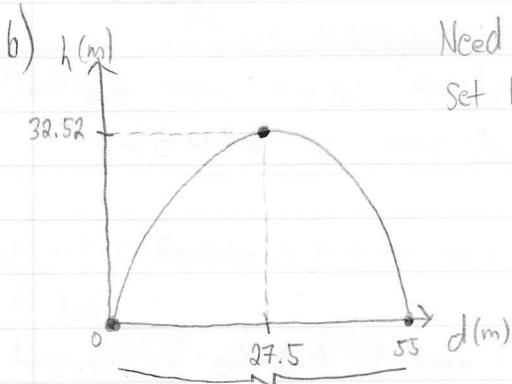
$$h = -0.043[(d^2 - 55d + 756.25) - 756.25]$$

$$h = -0.043(d^2 - 55d + 756.25) + 32.51875 \quad d(\text{m}) \quad h(\text{m})$$

$$h = -0.043(d - 27.5)^2 + 32.51875 \Rightarrow V(27.5, 32.51875)$$

\therefore The max height (to the nearest one hundredth of a meter is
32.52 m

(like x-intercepts)



Need d -intercepts to find width of base,

$$\text{Set } h = 0, 0 = -0.043(d - 27.5)^2 + 32.51875$$

$$\pm \sqrt{\frac{-32.51875}{-0.043}} = d - 27.5$$

$$27.5 \pm 27.5 = d$$

$$\therefore d = 0 \text{ m} \quad \& \quad d = 55 \text{ m}$$

$$(0,0) \quad \& \quad (55,0)$$

\therefore The width of the base is 55 m