MHF4U: Test – Functions: Understanding Rates of Change sept 2014

Name: _____ Mark: KU APP

1. Given $f(x) = -3x + 15$ and $g(x) = 2x^2 - 6$, evaluate or simplify each of the following: [KU/7]

(a) $-2f(-2) - g(\frac{1}{2})$

$= -2(21) - (-\frac{11}{2})$

$= -36.5$

(b) $f^{-1}(-3)$

$f^{-1}(x) = \frac{-x+15}{3}$

$f^{-1}(-3) = \frac{18}{3}$

$= 6$

Aside

INV $x = -3y + 15$

$y = \frac{-x+15}{3}$

(c) If $f(-5a) = -2a^2 - 10$, then determine the value(s) of a

$-3(-5a) + 15 = -2a^2 - 10$

$15a + 15 = -2a^2 - 10$

$2a^2 + 15a + 25 = 0$

$(2a+5)(a+5) = 0$

$\therefore a = -\frac{5}{2}$ $a = -5$

2. If $f(x) = -\left|\frac{1}{2}x - 3\right| + 4$ then,

A) Sketch both $f(x)$ and $f^{-1}(x)$ in the space provided. [KU/6]

$y = -\left|\frac{1}{2}(x-6)\right| + 4$

$(x, y) \rightarrow (2x+6, -y+4)$

BASIC

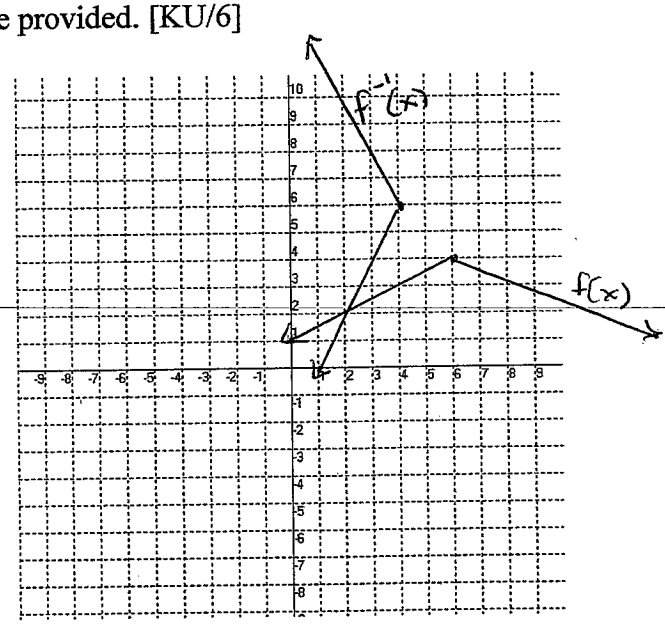
x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

f(x)

x	y
0	1
2	2
4	3
6	4
8	3
10	2
12	1

INVERSE

x	y
1	0
2	2
3	4
4	6
3	8
2	10
1	12



B) From the graph above, determine the domain and range of $f^{-1}(x)$. [KU/2]

$f^{-1}(x)$

$D = \{x \mid x \leq 4, x \in R\}$

$R = \{y \mid y \in R\}$

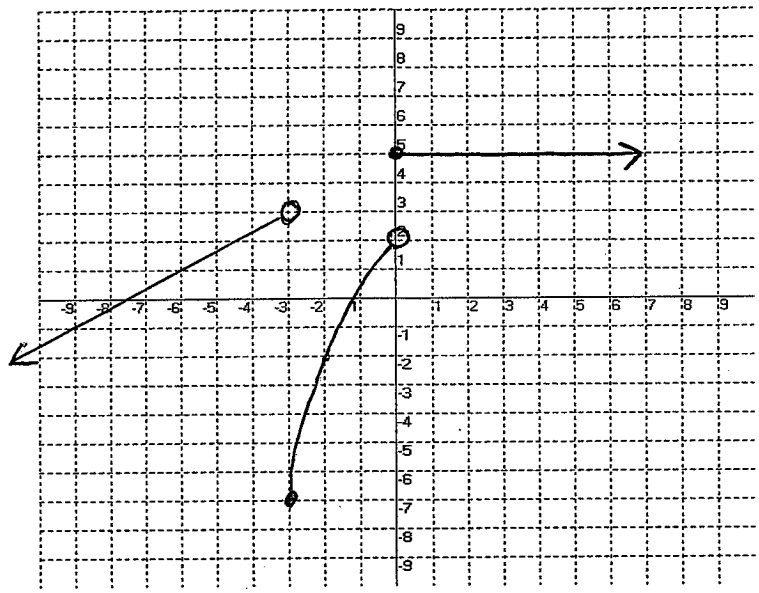
3. Sketch the piecewise function $f(x)$ on the graph given: [APP/8] $f(x)=\begin{cases}\frac{2}{3}x+5, x<-3\\ -x^2+2, -3\le x<0\\ 5, x\ge 0\end{cases}$

(a) Determine $f(-6)$ = 1

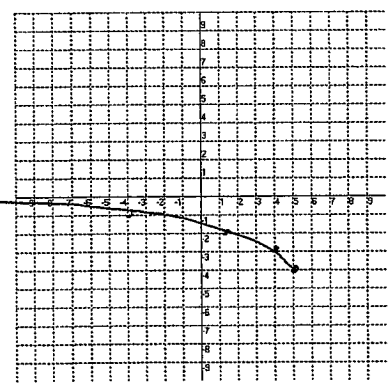
(b) Determine $f(-3)$ = -7

(c) Determine $f(0)$ = 5

(d) Determine $f(2)$ = 5



4. Sketch the following function in the space provided. [APP/6]

Equation	Transformation/Mapping	Graph
$y=\sqrt{5-x}-4$ $y=\sqrt{-x+5}-4$ $y=\sqrt{-(x-5)}-4$	$(x,y)\to(-x+5, y-4)$ x | y: 0 | 0, 1 | 1, 4 | 2, 9 | 3, 16 | 4 5 | -4, 4 | -3, 1 | -2, -4 | -1, -11 | 0 • reflection y-axis • right 5 • down 4	

5. During the 1997 Olympics in Greece, Donovan Bailey set the world record for a 100 m race. The table below shows Donovan's performance during the race. [KU/4]

Bailey's Race Performance	
Time(s) (x)	Distance (m) (y)
0	0
1.78	10
2.81	20
3.72	30
4.59	40
5.44	50
6.29	60
7.14	70
8.00	80
8.87	90
9.77	100

A) Calculate Donovan Bailey's average velocity for the second half of the race, $d \in [50,100]$ [2]

y	50	100
x	5.44	9.77

Avg ROC $= \frac{100-50}{9.77-5.44}$

$= 11.55$ m/s

B) Determine his instantaneous velocity at 50 metres. [2]

SECANT #1

y	40	50
x	4.59	5.44

Avg ROC = 11.76 m/s

SECANT #2

y	50	60
x	5.44	6.29

Avg ROC = 11.76 m/s

Inst ROC $= \frac{11.76 + 11.76}{2}$

$= 11.76$ m/s

6. A) What is the average rate of change for the functions $f(x) = x^3 - x^2 + 8x$ over the interval $-4 \le x \le -1$? Is the function increasing or decreasing over this interval? [KU/3]

y	-112	-10
x	-4	-1

Avg ROC $= \frac{-10-(-112)}{-1-(-4)}$

$= 34$

function is increasing

B) Using the Centered-Interval Method, estimate the instantaneous rate of change of the function when $x = 2$. [KU/3]

y	18.449	21.651
x	1.9	2.1

Avg ROC $= \frac{3.202}{0.2}$

$= 16.01$

∴ Estimate of the instantaneous ROC is 16.

7. The path of an arrow shot by an archer is given by the function $h(t) = -5t^2 + 20t + 2$ where $h(t)$ is the height in metres (m) is and t is the time in seconds(s). [APP/9]

A) What is the maximum height reached by the arrow? (Hint: Complete the Square) [3]

$y = -5(t^2 - 4t) + 2$
$= -5(t^2 - 4t + 4 - 4) + 2$
$= -5(t-2)^2 + 22$

$\therefore$ max height is 22 m

B) If the centre of the target is located at a height of 1.7 m , how long after the arrow was released did it take to hit the centre of the target? [3]

$1.7 = -5(t-2)^2 + 22$
$4.06 = (t-2)^2$
$t = 2 \pm 2.01$
$\therefore t = 4.01$ seconds

OR

$1.7 = -5t^2 + 20t + 2$
$0 = -5t^2 + 20t + 0.3$
quad formula
$t = 4.01$ seconds

C) Using rate of change, prove there is a max value at t=2 seconds. [3]

y	22	21.999995
x	2	2.001

Avg ROC = -0.005

$\therefore$ estimate of Instantaneous rate of change is 0

$\therefore$ max since slope of tangent @ 2 is 0

8. An automobile enters a road and travels the following distance in metres during the next 4 seconds, where s represents the distance in metres and t time in seconds.

t	0	1	2	3	4
s	0	5	14	27	44

If the points above satisfy the equation $s = at^2 + bt$, determine a and b then calculate the speed of the car when $t = 10$ seconds. (Use any Method) [APP 6]

sub in (1, 5)
① $5 = a(1)^2 + b(1)$

sub in (2, 14)
② $14 = a(2)^2 + b(2)$

① $a + b = 5$
② $4a + 2b = 14$

① $4a + 4b = 20$
② $4a + 2b = 14$
$2b = 6$
$b = 3$
$\therefore a = 2$

$\therefore$ $s = 2t^2 + 3t$

Inst ROC @ t = 10

y	230	230.043002
x	10	10.001

Avg ROC = 43.002

$\therefore$ estimate of Inst ROC is 43 m/s.