Polynomial Functions in Factored Form vs Standard Form

<table>
<thead>
<tr>
<th>STANDARD FORM (expanded form)</th>
<th>FACTORED FORM</th>
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<tbody>
<tr>
<td>( y = x^6 + 4x^5 - 23x^4 - 38x^3 + 220x^2 - 200x )</td>
<td>( y = (x)(x - 2)^3(x + 5)^2 )</td>
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<tr>
<td>- y-intercept is zero ((0,0))</td>
<td>- x intercepts at 0,2,-5 ((0,0)) ((2,0)) ((-5,0))</td>
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<td>- degree is 6</td>
<td>- “point of inflection” at (x=2)</td>
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<td>- positive lead coefficient (end behaviour)</td>
<td>- “turning point” or “touch point” at (x=-5)</td>
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<td>- Cross through point at (x=0)</td>
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A **family** of functions is a group of polynomial functions that would have the same **x-intercepts**. For example:

\[
  f(x) = (x - 2)^2(x + 3)(x - 4)
\]

\[
  f(x) = -3(x - 2)^2(x + 3)(x - 4)
\]

\[
  f(x) = 2(x - 2)^2(x + 3)(x - 4)
\]

\[
  f(x) = -(x - 2)^2(x + 3)(x - 4)
\]

**Let’s Try!**

Example 1: Determine the equation of a cubic polynomial function with zeros at -1,4,5 and a y-intercept of -40.

Example 2: Write a possible equation for a polynomial function with zeros at -3 \(\text{order} 2\) and \(\frac{1}{2}\).

Example 3: Sketch a possible graph of the polynomial graph

\[
  f(x) = -(x - 4)(2x - 5)(x + 1)^2
\]

Example 4: Sketch a possible graph of the polynomial graph

\[
  f(x) = 3x^4 - 3x^3
\]

Example 5: Determine a possible polynomial function with roots 2 , -4 and 5/4.

Example 6: Determine a general equation for the cubic family of functions with x-intercepts at -3, -4, 5. Then determine the particular equation of the family that passes through the point \((3, -252)\).

Example 7: Determine a general equation for the cubic family of functions with x-intercepts at -3, 0, 2. Then determine the particular equation of the family that passes through the point \((-1, 12)\).

Example 8: Determine a general equation for the cubic family of functions with x-intercepts at -2, -1, 1. Then determine the particular equation of the family that passes through the point \((2, -6)\).

Example 9: Determine a general equation for the quartic family of functions with x-intercepts at -2, -1, 1, 3. Then determine the particular equation of the family that passes through the point \((2, -6)\).