

Inverse of Functions - Let's Look at the Equations!

To determine the inverse graph of a function, we have learned that we simply need to "switch" the x-values with the y-values.

To determine the inverse equation of a function, we need to do the exact same thing!

Determine the inverse equation for each of the following:

$$1) y = x + 5$$

$$\text{Inv } x = y + 5$$

$$x - 5 = y$$

$$y = x - 5$$

$$2) y = 2x - 1$$

$$\text{Inv } x = 2y - 1$$

$$x + 1 = 2y$$

$$\frac{x+1}{2} = y$$

$$y = \frac{x+1}{2}$$

$$3) y = \frac{3-x}{2}$$

$$\text{Inv } x = \frac{3-y}{2}$$

$$2x = 3 - y$$

$$2x - 3 = -y$$

$$y = -2x + 3$$

$$4) y = \frac{5}{x} + 4$$

$$\text{Inv } x = \frac{5}{y} + 4$$

$$x - 4 = \frac{5}{y}$$

$$y(x-4) = y\left(\frac{5}{y}\right)$$

$$\frac{y(x-4)}{(x-4)} = \frac{5}{(x-4)}$$

$$6) f(x) = 4 - 3x^2$$

$$\text{Inv } y = 4 - 3x^2$$

$$\text{Inv } x = 4 - 3y^2$$

$$x - 4 = -3y^2$$

$$\frac{x-4}{-3} = y^2$$

$$\frac{-x+4}{3} = y^2$$

$$\pm \sqrt{\frac{-x+4}{3}} = y$$

$$\therefore f^{-1}(x) = \pm \sqrt{\frac{-x+4}{3}}$$

of $f(x)$.



Now let's combine this concept of finding inverse equations with what we already know about function notation!

Given:

$$f(x) = 2x + 4 \quad g(x) = 3x^2 + 1 \quad h(x) = \frac{x+2}{5}$$

Determine each of the following:

(a) $f(4)$

$$f(x) = 2x + 4$$

$$f(4) = 2(4) + 4$$

$$f(4) = 12$$

\therefore the pt $(4, 12)$ is on the line.

(c) $g(h(8))$

$$h(8) = \frac{x+2}{5}$$

$$h(8) = \frac{8+2}{5}$$

$$h(8) = 2$$

$$g(h(8)) = 3(2)^2 + 1$$

$$g(h(8)) = 13$$

(e) $f(h^{-1}(x))$

$$f(h^{-1}(x)) = 2(5x-2) + 4$$

$$f(h^{-1}(x)) = 10x - 4 + 4$$

$$f(h^{-1}(x)) = 10x$$

Side
 $y = \frac{x+2}{5}$

Inv
 $x = \frac{y+2}{5}$

$$5x = y + 2$$

$$5x - 2 = y$$

$$h^{-1}(x) = 5x - 2$$

(b) $f^{-1}(3)$

$$f^{-1}(3) = \frac{3-4}{2}$$

$$f^{-1}(3) = -\frac{1}{2}$$

\therefore the pt $(-\frac{1}{2}, 3)$

Side

$$y = 2x + 4$$

Inv

$$x = 2y + 4$$

$$x - 4 = 2y$$

$$\frac{x-4}{2} = y$$

$$f^{-1}(x) = \frac{x-4}{2}$$

(d) $g^{-1}(h(+8))$

$$g^{-1}(2) = \pm \sqrt{\frac{x-1}{3}}$$

$$g^{-1}(2) = \pm \sqrt{\frac{2-1}{3}}$$

$$g^{-1}(2) = \pm \sqrt{\frac{1}{3}}$$

$$g^{-1}(2) = \pm \sqrt{\frac{1}{\sqrt{3}}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$g^{-1}(2) = \pm \frac{\sqrt{3}}{3}$$

Side

$$y = 3x^2 + 1$$

Inv

$$x = \sqrt{y-1}$$

$$x - 1 = 3y^2$$

$$\frac{x-1}{3} = y^2$$

$$\pm \sqrt{\frac{x-1}{3}} = y$$

$$g^{-1}(x) = \pm \sqrt{\frac{x-1}{3}}$$

(f) $g(f^{-1}(5))$

$$g\left(\frac{1}{2}\right) = 3x^2 + 1$$

$$g\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 + 1$$

$$g\left(\frac{1}{2}\right) = \frac{3}{4} + 1$$

$$g\left(\frac{1}{2}\right) = \frac{7}{4}$$

Side

$$y = 2x + 4$$

Inv

$$x = \frac{y-4}{2}$$

$$x - 4 = 2y$$

$$\frac{x-4}{2} = y$$

$$f^{-1}(x) = \frac{x-4}{2}$$

$$(f^{-1}(5)) = \frac{5-4}{2}$$

$$(f^{-1}(5)) = \frac{1}{2}$$