

## Inverse of a Function

switch  
+ values  
; y values

An inverse of a function can be obtained by:

- 1) mapping  $(x,y) \rightarrow (y,x)$
- 2) reflecting the graph of the function over the line  $y=x$

### Example 1

("points")

Given  $f = \{(2,3), (5,9), (7,1)\}$ , find the inverse.

State the domain and range of the  $f$  and of the  $f^{-1}$ .

$$f = \{(2,3), (5,9), (7,1)\} \quad D = \{x | x \in \{2, 5, 7\}\} \quad R = \{y | y \in \{1, 3, 9\}\}$$

↓ Domain  
↓ Range  
↓ Switch

*Inverse function*  $f^{-1} = \{(3,2), (9,5), (1,7)\} \quad D = \{x | x \in \{1, 3, 9\}\} \quad R = \{y | y \in \{2, 5, 7\}\}$

### Example 2

("graph")

- (a) Graph the function  $f(x) = (x+3)^2 + 1$  and also graph the inverse function.
- (b) State the domain and range of  $f(x)$  and of  $f^{-1}(x)$ .
- (c) Is the inverse also a function?
- (d) Restrict the domain of  $f(x)$  so that the inverse is a function.

### Example 3

("equations")

Find the inverse equation of each of the following functions:

- (a)  $f(x) = 3x + 4$
- (b)  $g(x) = x^3 - 1$
- (c)  $h(x) = -x$
- (d)  $m(x) = -2(x+5)$

$$2. \quad f(x) = (x+3)^2 + 1$$

$$(x, y) \rightarrow (x-3, y+1)$$

$y = x^2$	X	Y
-3	9	
-2	4	
-1	1	
0	0	
1	1	
2	4	
3	9	

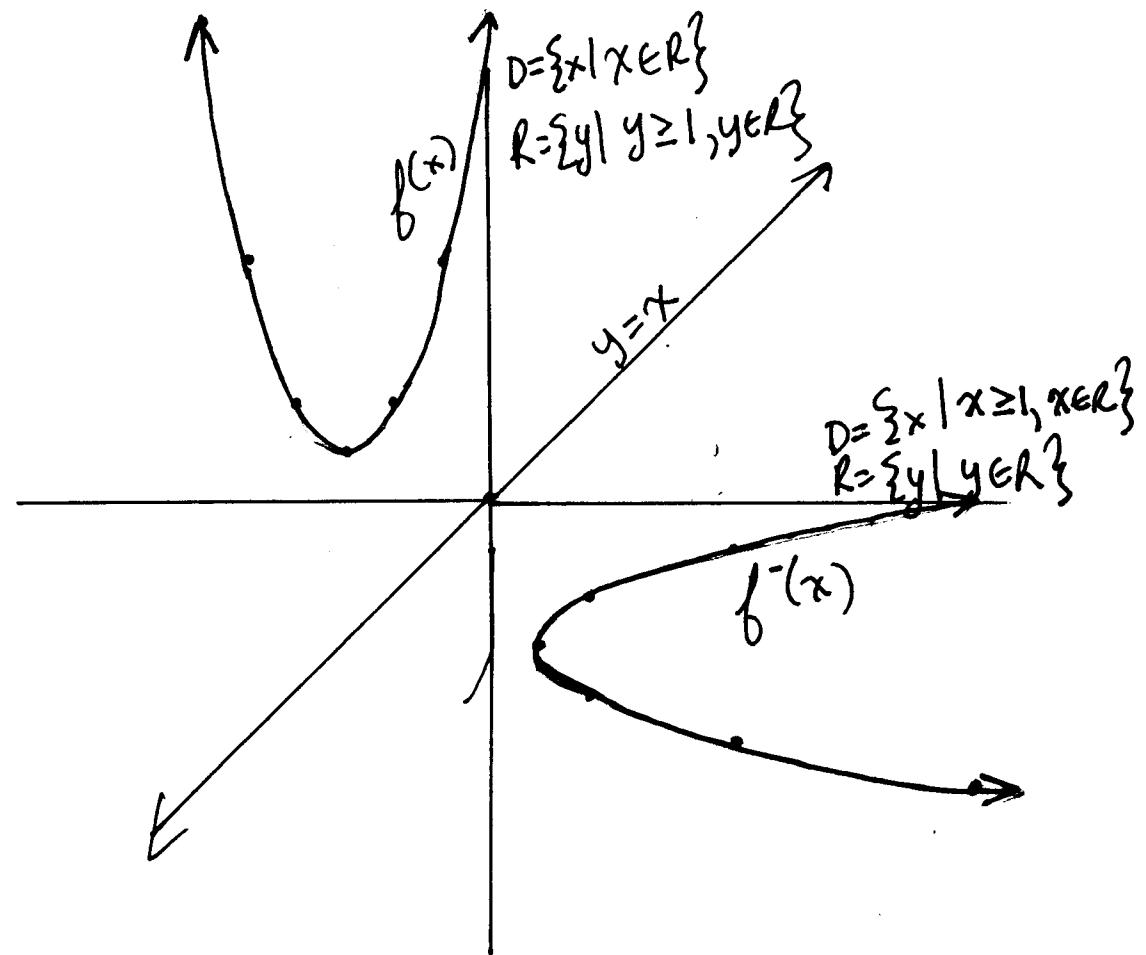
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$y = (x+3)^2 + 1$	X	Y
-6	10	
-5	5	
-4	2	
-3	1	
-2	2	
-1	5	
0	10	

"the function"

$x$	$y$
10	-6
5	-5
2	-4
1	-3
2	-2
5	-1
10	0

"the inverse"



$$(a) f(x) = 3x + 4$$

$$y = 3x + 4$$

INV

$$x = 3y + 4 \quad \text{①}$$

$$x - 4 = 3y$$

$$\frac{x-4}{3} = y$$

$$f^{-1}(x) = \frac{x-4}{3}$$

$$(b) g(x) = x^3 - 1$$

$$y = x^3 - 1$$

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$$x = y^3 - 1$$

$$x + 1 = y^3$$

$$\sqrt[3]{x+1} = y$$

$$g^{-1}(x) = \sqrt[3]{x+1}$$

$$f^{-1}(x) = -x$$

$$m^{-1}(x) = \frac{-x}{2} - 5 \quad \checkmark$$

$$m^{-1}(x) = \frac{-x-10}{2} \quad \checkmark$$