

Inverse of Functions - Let's Look at the Equations!

To determine the inverse graph of a function, we have learned that we simply need to “switch” the x-values with the y-values.

To determine the inverse equation of a function, we need to do the exact same thing!

Determine the inverse equation for each of the following:

$$1) \quad y = x + 5$$

$$2) \quad y = 2x - 1$$

$$3) \quad y = \frac{3-x}{2}$$

$$4) \quad y = \frac{5}{x} + 4$$

$$5) \quad y = (x + 2)^2$$

$$6) \quad f(x) = 4 - 3x^2$$

Now let's combine this concept of finding inverse equations with what we already know about function notation!

Given:

$$f(x) = 2x + 4 \quad g(x) = 3x^2 + 1 \quad h(x) = \frac{x+2}{5}$$

Determine each of the following:

(a) $f(4)$

(b) $f^{-1}(3)$

(c) $g(h(8))$

(d) $g^{-1}(h(-7))$

(e) $f(h^{-1}(x))$

(f) $g(f^{-1}(5))$

Find the Inverse equation of each function!

$$\textcircled{1} \quad f(x) = 2x + 4$$

$$y = 2x + 4$$

[INV]

$$x = 2y + 4$$

$$\frac{x-4}{2} = y$$

$$\therefore f^{-1}(x) = \frac{x-4}{2}$$

$$\textcircled{2} \quad g(x) = 3x^2 + 1$$

$$y = 3x^2 + 1$$

[INV]

$$x = 3y^2 + 1$$

$$x - 1 = 3y^2$$

$$\frac{x-1}{3} = y^2$$

$$g^{-1}(x) = \pm \sqrt{\frac{x-1}{3}}$$

$$y = \pm \sqrt{\frac{x-1}{3}}$$

$$\textcircled{3} \quad f(x) = \frac{x+2}{5} \quad y = \frac{x+2}{5}$$

$$[INV] \quad x = \frac{y+2}{5}$$

$$f^{-1}(x) = 5x - 2$$

$$5x - 2 = y$$

Now let's combine this concept of finding inverse equations with what we already know about function notation!

Solutions

Given:

$$f(x) = 2x + 4 \quad g(x) = 3x^2 + 1 \quad h(x) = \frac{x+2}{5}$$

Determine each of the following:

$$\begin{aligned} \text{(a)} \quad f(4) &= 2(4) + 4 \\ &= 8 + 4 \\ &= 12 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad f^{-1}(3) &= \frac{3-4}{2} \\ &= -\frac{1}{2} \end{aligned}$$

$$\begin{array}{l|l} \text{(c)} \quad g(h(8)) & \\ \hline h(8) = \frac{8+2}{5} & \left| \begin{array}{l} g(z) = 3(z)^2 + 1 \\ = 13 \end{array} \right. \\ = 2 & \end{array}$$

$$\begin{aligned} \text{(d)} \quad g^{-1}(h(-7)) & \\ h(-7) = \frac{-7+2}{5} & \\ = -1 & \left| \begin{array}{l} g^{-1}(-1) \\ = \pm \sqrt{\frac{-1-1}{3}} \\ = \pm \sqrt{-\frac{2}{3}} \\ \therefore \text{undefined} \end{array} \right. \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad f(h^{-1}(x)) & \\ = f(5x-2) & \\ = 2(5x-2) + 4 & \\ = 10x - 4 + 4 & \\ = 10x & \end{aligned}$$

$$\begin{aligned} \text{(f)} \quad g(f^{-1}(5)) & \\ f^{-1}(5) = \frac{5-4}{2} & \\ = \frac{1}{2} & \left| \begin{array}{l} g\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 + 1 \\ = \frac{3}{4} + 1 \\ = \frac{7}{4} \end{array} \right. \end{aligned}$$