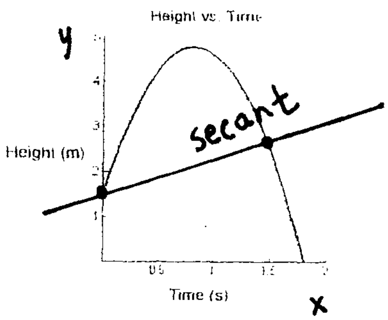
Instantaneous Rate of Change

Example #1

The height of a shot put can be modeled by the equation: $y = h(x) = -4.9x^2 + 8x + 1.5$
Where x is the time in seconds and h(x) is the height in metres.

(a) Find the average rate of change for the interval $x \in [0,1.5]$

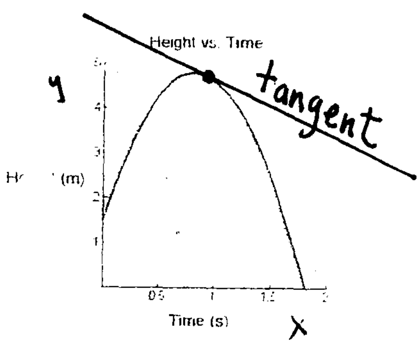


y	1.5	2.475
x	0	1.5

$$\text{Avg ROC} = \frac{2.475-1.5}{1.5-0} = \boxed{0.65 \text{ m/s}}$$

The slope of a secant line is the average rate of change.

(b) Find the instantaneous rate of change at the value $x = 1$.

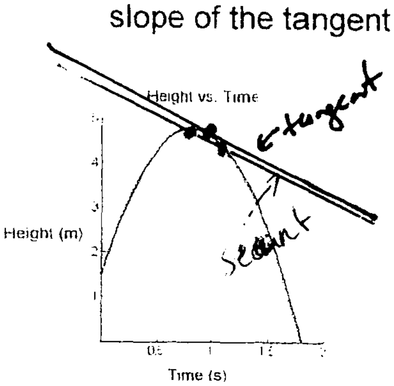


* the slope of the tangent line will give us the instantaneous rate of change at that point!

There are 3 methods we could use to find the instantaneous rate of change:

Method 1: Centred Interval Method

For this method, we will get the average rate of change between two points that are on either side of the target point. The two points should be equidistant and not too far from the target point. The idea is that the slope of this secant line will approximate the actual slope of the tangent line at the target.



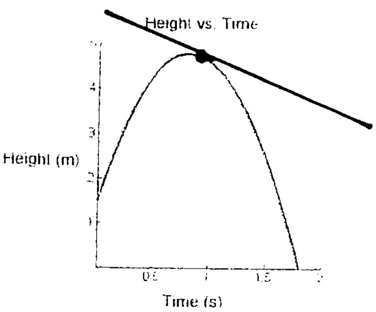
y	4.731	4.371
x	0.9	1.1

x=1 target

$$\text{Avg ROC} = \frac{4.371-4.731}{1.1-0.9} = \boxed{-1.8 \text{ m/s}}$$

∴ we estimate the instantaneous R.o.C. @ x=1 to be -1.8 m/s

Method 2: Difference Quotient Method
For this method, we will get the average rate of change between the target point and another point very, very, very close to the target point. The idea is that this secant line will approximate very closely the tangent line at the target point.



Y	4.6	4.5981951
x	1	1.001

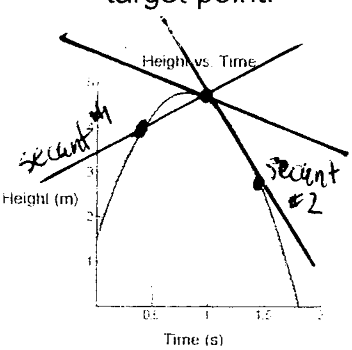
target $x=1$

$$\text{Avg ROC} = \frac{4.5981951 - 4.6}{1.001 - 1}$$

$$= -1.8049 \text{ m/s}$$

∴ this is an approximation of the instantaneous ROC at $x=1$

Method 3: Preceding and Following Method
For this method, we will get two average rates of change on either side of the target point. One secant line will pass through the target point and a point that precedes it. The other secant line will pass through the target point and a point that follows it. The two points should be equidistant from the target point. We will then take an average of the slopes of the two secant lines to approximate the actual slope of the tangent line at the target point.



Secant #1
preceding

Y	4.275	4.6
x	0.5	1

$$\text{Avg R.O.C} = \frac{4.6 - 4.275}{1 - 0.5}$$

$$= 0.65 \text{ m/s}$$

Secant #2
following

Y	4.6	2.475
x	1	1.5

$$\text{Avg R.O.C} = \frac{2.475 - 4.6}{1.5 - 1}$$

$$= -4.25 \text{ m/s}.$$

Approx. for instantaneous $= \dfrac{0.65 + (-4.25)}{2}$

$= -1.8 \text{ m/s}$

Example #2
An automobile enters a road and travels the following distances in metres during the next 6 seconds, where s represents the distance in metres and t represents time in seconds:

	t	0	1	2	3	4	5	6
x	s	0	1	3	6	10	15	21

Y

Find the instantaneous rate of change at $t = 2$.

Method 1

Y		
x	1	3

$x=2$

① W/S
② Text pg 85 # 1-5, 7, 9, 10

Method 3

Y		
X	1	2

preceding

Y		
X	2	3

following