



$$\left(\frac{S_n = a(r^n - 1)}{r - 1} \right)$$

GEOMETRIC SERIES ($t_n = a(r)^{n-1}$)

Geometric Series – the sum of the terms of a geometric sequence

- examples
- 1) $1+2+4+8+16+\dots$ $a=1$ $r=2$
 - 2) $2+10+50+250+\dots$ $a=2$ $r=5$
 - 3) $-3+6-12+24+\dots$ $a=-3$ $r=-2$

S_n formula for each of the geometric series examples

$$\begin{aligned} 1) \quad t_n &= 2^{n-1} \\ S_n &= \frac{a(r^n - 1)}{r - 1} \\ &= \frac{1(2^n - 1)}{2 - 1} \\ &= 2^n - 1 \end{aligned}$$

$$\begin{aligned} 2) \quad t_n &= 2(5)^{n-1} \\ S_n &= \frac{2(5^n - 1)}{5 - 1} \\ &= \frac{2(5^n - 1)}{4} \\ &= \frac{5^n - 1}{2} \end{aligned}$$

$$\begin{aligned} 3) \quad t_n &= -3(-2)^{n-1} \\ S_n &= \frac{-3(-2^n - 1)}{-2 - 1} \\ &= \frac{-3(-2^n - 1)}{-3} \\ &= (-2)^n - 1 \end{aligned}$$

find t_7 and S_6 for each of the geometric series examples

$$\begin{aligned} 1) \quad t_7 &= 2^{7-1} \\ &= 2^6 \\ &= 64 \\ S_6 &= 2^6 - 1 \\ &= 128 - 1 \\ &= 127 \end{aligned}$$

$$\begin{aligned} 2) \quad t_7 &= 2(5)^{7-1} \\ &= 2(15625) \\ &= 31250 \\ S_6 &= \frac{5^6 - 1}{2} \\ &= \frac{15624}{2} \\ &= 7812 \end{aligned}$$

$$\begin{aligned} 3) \quad t_7 &= -3(-2)^{7-1} \\ &= -3(64) \\ &= -192 \\ S_6 &= (-2)^6 - 1 \\ &= 64 - 1 \\ &= 63 \end{aligned}$$

Ex.1 Find the sum of the following series

$$a = -2 \quad r = 3$$

(a) $-2 - 6 - 18 - 54 - \dots - 4374$

$$\begin{aligned} t_n &= -2(3)^{n-1} & S_6 &= -(3^8 - 1) \\ -4374 &= -2(3)^{n-1} & &= -(6560) \\ 2187 &= 3^{n-1} & &= -6560 \\ 37 &= 3^{n-1} \end{aligned}$$

$$\therefore n = 8$$

$$\begin{aligned} S_n &= \frac{-2(3^n - 1)}{3 - 1} \\ &= -(3^n - 1) \end{aligned}$$

$$a = \frac{1}{4} \quad r = 2$$

(b) $\frac{1}{4} + \frac{1}{2} + 1 + 2 + 4 + \dots + 512$

$$\begin{aligned} t_n &= \frac{1}{4}(2)^{n-1} & S_{12} &= \frac{1}{4}(2^{12} - 1) \\ 512 &= \frac{1}{4}(2)^{n-1} & &= \frac{1}{4}(4095) \\ 2048 &= 2^{n-1} & &= 1023.75 \\ 2^{11} &= 2^{n-1} \end{aligned}$$

$$\therefore n = 12$$

$$\begin{aligned} S_n &= \frac{\frac{1}{4}(2^n - 1)}{2 - 1} \\ &= \frac{1}{4}(2^n - 1) \end{aligned}$$