FINITE DIFFERENCES

Numerical Properties of Polynomial Functions

1. Consider the function $y = x$
   a) What type of function is it?
   b) Complete the table of values.
   c) Calculate the first differences.
   d) In this case, the first differences were positive. How would the graph differ if the first differences were negative?)

Linear

$\boxed{y = x}$

x	y	First Differences
−3	−3	
		▶ +1
−2	−2	
		▶ +1
−1	−1	
		▶ +1
0	0	
		▶ +1
1	1	
		▶ +1
2	2	
		▶ +1
3	3	

First Difference is constant ∴ it is linear

2. Consider the function $y = x^2$
   a) What type of function is it?
   b) Complete the table of values.
   c) Calculate the first and second differences.

Quadratic

$\boxed{y = x^2}$

x	y	Differences: First Δy	Differences: Second $\Delta^2 y$
−3	9		
		▶ −5	
−2	4		▶ +2
		▶ −3	
−1	1		▶ +2
		▶ −1	
0	0		▶ +2
		▶ +1	
1	1		▶ +2
		▶ +3	
2	4		▶ +2
		▶ +5	
3	9		

2nd difference is constant ∴ it is a quadratic

3. Consider the function $y = x^3$
   a) What type of function is it?
   b) Complete the table of values.
   c) Calculate the first, second, and third differences.

Cubic

$\boxed{y = x^3}$

x	y	Differences: First Δy	Differences: Second $\Delta^2 y$	Differences: Third $\Delta^3 y$
−3	−27			
		▶ +19		
−2	−8		▶ −12	
		▶ +7		▶ +6
−1	−1		▶ −6	
		▶ +1		▶ +6
0	0		▶ 0	
		▶ +1		▶ +6
1	1		▶ 6	
		▶ +7		▶ +6
2	8		▶ 12	
		▶ +19		
3	27			

3rd difference is constant ∴ it is a cubic

4. Consider the function $y = x^4$
   a) What type of function is it?
   b) Complete the table of values.
   c) Calculate the first, second, and third differences.

x	y	Differences: First Δy	Second $\Delta^2 y$	Third $\Delta^3 y$	Fourth $\Delta^4 y$
−3	81				
		▶ −65			
−2	16		▶ 50		
		▶ −15		▶ −36	
−1	1		▶ 14		▶ 24
		▶ −1		▶ −12	
0	0		▶ 2		▶ 24
		▶ 1		▶ 12	
1	1		▶ 14		▶ 24
		▶ 15		▶ 36	
2	16		▶ 50		
		▶ 65			
3	81				

Quartic

$\boxed{y = x^4}$

4th difference is constant ∴ it is a quartic

Summary

$\boxed{y = x}$	linear	$\Delta y = 1$
$\boxed{y = x^2}$	quadratic	$\Delta^2 y = 2$
$\boxed{y = x^3}$	cubic	$\Delta^3 y = 6$
$\boxed{y = x^4}$	quartic	$\Delta^4 y = 24$