# Determining Equations of Polynomials Given Points on the Function

For each of the following sets of points, determine the equation of the polynomial function.

*Example 1*

(1,0) (2,3) (3,10) (4,21) (5,36) (6,55)

| x | y | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| 1 | 0 | | | |
| | | +3 | | |
| 2 | 3 | | +4 | |
| | | +7 | | |
| 3 | 10 | | +4 | |
| | | +11 | | |
| 4 | 21 | | +4 | |
| | | +15 | | |
| 5 | 36 | | +4 | |
| | | +19 | | |
| 6 | 55 | | | |

$\therefore$ our function is a quadratic

for $y = x^2$, $\Delta^2 y = 2$
for ours, $\Delta^2 y = 4$ $\times 2$

$\therefore a = 2$

$y = ax^2 + bx + c$

$\therefore y = 2x^2 + bx + c$

sub (1,0)
$0 = 2(1)^2 + b(1) + c$
① $\boxed{b + c = -2}$

sub (2,3)
$3 = 2(2)^2 + b(2) + c$
② $\boxed{2b + c = -5}$

$b + c = -2$
$2b + c = -5$
$-b = 3$
$\therefore b = -3$ $\therefore c = 1$

$\boxed{\therefore y = 2x^2 - 3x + 1}$

*Example 2*

(1,-3) (2,2) (3,33) (4,108) (5,245) (6,462)

| x | Y | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| 1 | -3 | | | |
| 2 | 2 | | | +18 |
| 3 | 33 | | | +18 |
| 4 | 108 | | | |
| 5 | 245 | | | |
| 6 | 462 | | | |

$\therefore$ our function is a cubic

for $y = x^3$, $\Delta^3 y = 6$
for ours, $\Delta^3 y = 18$ $\times 3$

$\therefore a = 3$

$y = ax^3 + bx^2 + cx + d$

$y = 3x^3 + bx^2 + cx + d$

sub in (1,-3) sub in (2,2) sub (3,33)

① $b + c + d = -6$

② $4b + 2c + d = -22$

③ $9b + 3c + d = -48$

① $b+c+d=-6$ ② $4b+2c+d=-22$ ③ $9b+3c+d=-48$

eliminate d

① $b+c+d=-6$
② $4b+2c+d=-22$

subt $-3b-c=16$ ④

eliminate d

② $4b+2c+d=-22$
③ $9b+3c+d=-48$

subt $-5b-c=26$ ⑤

④ $-3b-c=16$
⑤ $-5b-c=26$

subt $2b=-10$

$b=-5$

$\therefore c=-1$

$\therefore d=0$

$$\therefore y=3x^3-5x^2-1x$$