

# MHF 4U

## FINAL EXAM

### REVIEW (SAMPLER)

#### Section A – RATES of CHANGE

(1) Determine the average rate of change for the following function:

$$f(x) = -3x^5 + 2x^3 - 4$$

$$-2 \leq x \leq 1$$

(2) Determine the average rate of change for the following function:

$$f(x) = 5x^3 - x^2 + 2x$$

$$x \in (3,4)$$

(3) Determine the instantaneous rate of change using the centered interval method for the following function:

$$f(x) = -3x^3 + 2x^2 - 4$$

$$@x = -5$$

(4) Determine the instantaneous rate of change using the difference quotient method for the following function:

$$f(x) = -x^2 + 2x$$

$$@x = 3$$

#### Section B – POLYNOMIAL FUNCTIONS

(5) Determine the degree, the x-intercept(s), the y-intercept and end behaviours for each of the following polynomial functions

$$g(x) = -x(2x+1)^2(x-1)^3(x+3)$$

$$h(x) = 6x^3 - x^2 - 4x - 1$$

(6) Sketch each function from (5).

(7) When  $f(x) = 9x^2 - mx + k$  is divided by  $(x+4)$  the remainder is 164 and when it is divided by  $(x-2)$  the remainder is 38. Determine the remainder when  $f(x)$  is divided by  $(x-1)$ .

$$(8) \text{ Solve: } 4x^6 + 4x^5 = 24x^4$$

$$(9) \text{ Solve: } x^4 - 2x^3 - 13x^2 + 14x + 24 = 0$$

$$(10) \text{ Solve: } x^3 + 4x^2 + x - 6 < 0$$

#### Section C – RATIONAL FUNCTIONS

(11) Determine all discontinuities (asymptotes and holes) for each of the following:

$$f(x) = \frac{x}{x^3 - x}$$

$$g(x) = \frac{2x^2 - x - 1}{x^2 - x}$$

$$h(x) = \frac{x^3 + 5}{x^2 - 4x + 3}$$

(12) Determine the x-intercept(s) and y-intercept for  $f(x)$  and  $g(x)$ .

(13) Sketch a graph of  $f(x)$  and  $g(x)$ .

(14) Determine the point where  $h(x)$  crosses the asymptote.

$$(15) \text{ Solve: } \frac{x}{x-2} - \frac{3x}{x+1} = 5$$

$$(16) \text{ Solve: } \frac{x-1}{x+3} \leq \frac{3}{x-2}$$

#### Section D – TRIGONOMETRIC FUNCTIONS

$$(17) \text{ Determine } \sec\left(\frac{-5\pi}{4}\right)$$

(18) Determine exact primary trig ratios given that  $\cot \theta = \frac{2}{5}$  and the terminal arm is in Quad III. Also determine the measure of angle theta.

(19) Determine  $\cos \theta$  given that the terminal arm passes through the point  $(4, -2)$ . Also determine measure of angle theta.

(20) Evaluate the exact trig ratio for  $\sin\left(\frac{-5\pi}{12}\right)$

(21) Evaluate the exact trig ratio for  $1 - 2 \sin^2\left(\frac{7\pi}{8}\right)$

(22) The angle  $\angle \theta$  lies in the second quadrant such that  $\cos \theta = -\frac{11}{17}$ . Determine the exact value of  $\sin(2\theta)$ . Also determine the measure of  $\angle 2\theta$ .

(23) Solve the following trigonometric equation for all real roots over the specified interval.

$$3 \cos x \sec x - 7 \cos x = 0, \quad x \in [0, 2\pi]$$

(24) Solve the following trigonometric equation for all real roots over the specified interval.

$$-2 \cos 2x = 4 \sin x + 1, \quad x \in [0, 2\pi]$$

#### Section E – EXPONENTIAL AND LOGARITHMIC FUNCTIONS

$$(25) \text{ Simplify: } \log 20 + \log 5$$

$$(26) \text{ Simplify: } 3 \log_{16} 4$$

$$(27) \text{ Simplify: } \log_3 12 - 2 \log_3 2$$

(28) Use log laws to express the following as a single log:

$$3(\log_4 x - \log_4 y) + 2 \log_4 \left(\frac{3y}{x}\right)$$

$$(29) \text{ Solve: } \log x - \log(x-1) = 2$$

$$(30) \text{ Solve: } 3^{2x} = 5^{x-1}$$

#### ANSWERS

- A (1) -27  
 (2) 180  
 (3) -245  
 (4) -4

- B (5) degree 7, x-int 0, -1/2, 1, -3, y-int 0  
 degree 3, x-int 1, -1/2, -1/3, y-int -1  
 (7) 14  
 (8) 0, -3, 2  
 (9) -1, 2, 4, -3  
 (10)  $x \in (-\infty, -3)$   
 $x \in (-2, 1)$

- C (11) f(x) horiz y=0, vert x=1, x=-1, hole (0,-1)  
 g(x) horiz y=2, vert x=0, hole (1,3)  
 h(x) oblique  $y=x+4$ , vert x=3, x=1  
 (12) f(x) (0,-1) and no x-intercept  
 g(x) no y-intercept and (-1/2, 0)  
 (14) (7/13, 59/13)  
 (15)  $\frac{6 \pm \sqrt{106}}{7}$  restrict  $x \neq 2, x \neq -1$   
 (16)  $x \in (-3, -1]$   
 $x \in (2, 7]$

- D (17)  $-\sqrt{2}$   
 (18)  $\sin \theta = -\frac{5\sqrt{29}}{29}, \cos \theta = -\frac{2\sqrt{29}}{29}, \tan \theta = \frac{5}{2}$   
 angle theta = 4.33 rads  
 (19)  $\cos \theta = \frac{2\sqrt{5}}{5}$   
 (20)  $\frac{-\sqrt{6} - \sqrt{2}}{4}$   
 angle theta = 5.82 rads  
 (21)  $\frac{\sqrt{2}}{2}$   
 (22)  $\sin 2\theta = -\frac{44\sqrt{42}}{289}, 2\theta = 4.55 \text{ rads}$   
 (23)  $x = \frac{\pi}{2}, \frac{3\pi}{2}, 1.13 \text{ rads}, 5.16 \text{ rads}$   
 (24)  $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

- E (25) 2  
 (26) 3/2  
 (27) 1  
 (28)  $\log_3\left(\frac{9x}{y}\right)$   
 (29)  $x=100/99$   
 (30)  $x=-2.74$