Average Rate of Change Textbook page 76 # 1-4,6,8-10

In Summary

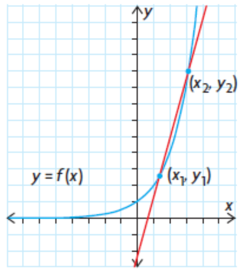
Key Ideas

- The average rate of change is the change in the quantity represented by the dependent variable (Δy) divided by the corresponding change in the quantity represented by the independent variable (Δx) over an interval. Algebraically, the average rate of change for any function $y = f(x)$ over the interval $x_1 \leq x \leq x_2$ can be determined by

$$\text{Average rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

- Graphically, the average rate of change for any function $y = f(x)$ over the interval $x_1 \leq x \leq x_2$ is equivalent to the slope of the secant line passing through two points (x_1, y_1) and (x_2, y_2).

$$\text{Average rate of change} = m_{\text{secant}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



Need to Know

- Average rate of change is expressed using the units of the two quantities that are related to each other.
- A positive average rate of change indicates that the quantity represented by the dependent variable is increasing on the specified interval, compared with the quantity represented by the independent variable. Graphically, this is indicated by a secant line that has a positive slope (the secant line rises from left to right).
- A negative average rate of change indicates that the quantity represented by the dependent variable is decreasing on the specified interval, compared with the quantity represented by the independent variable. Graphically, this is indicated by a secant line that has a negative slope (the secant line falls from left to right).
- All linear relationships have a constant rate of change. Average rate of change calculations over different intervals of the independent variable give the same result.
- Nonlinear relationships do not have a constant rate of change. Average rate of change calculations over different intervals of the independent variable give different results.

1. Calculate the average rate of change for the function $g(x) = 4x^2 - 5x + 1$ over each interval.

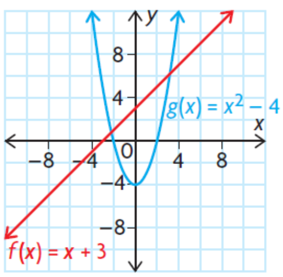
a) $2 \leq x \leq 4$ **d)** $2 \leq x \leq 2.25$
b) $2 \leq x \leq 3$ **e)** $2 \leq x \leq 2.1$
c) $2 \leq x \leq 2.5$ **f)** $2 \leq x \leq 2.01$

2. An emergency flare is shot into the air. Its height, in metres, above the ground at various times in its flight is given in the following table.

Time (s)	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Height (m)	2.00	15.75	27.00	35.75	42.00	45.75	47.00	45.75	42.00

a) Determine the average rate of change in the height of the flare during each interval.
i) $1.0 \leq t \leq 2.0$ **ii)** $3.0 \leq t \leq 4.0$

b) Use your results from part a) to explain what is happening to the height of the flare during each interval.



3. Given the functions $f(x)$ and $g(x)$ shown on the graph, discuss how the average rates of change, $\frac{\Delta y}{\Delta x}$, differ in each relationship.

4. This table shows the growth of a crowd at a rally over a 3 h period.

K

Time (h)	0.0	0.5	1.0	1.5	2.0	2.5	3.0
Number of People	0	176	245	388	402	432	415

a) Determine the average rate of change in the size of the crowd for each half hour of the rally.

b) What do these numbers represent?

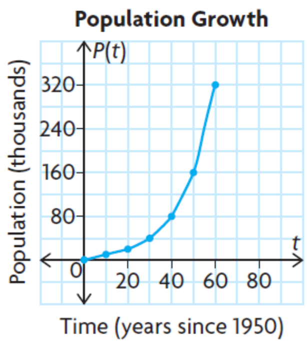
c) What do positive and negative rates of change mean in this situation?

6. What is the average rate of change in the values of the function $f(x) = 4x$ from $x = 2$ to $x = 6$? What about from $x = 2$ to $x = 26$? What do your results indicate about $f(x)$?

8. The population of a city has continued to grow since 1950. The population P, in thousands, and the time t, in years, since 1950 are given in the table below and in the graph.

A

Time, t (years)	0	10	20	30	40	50	60
Population, P (thousands)	5	10	20	40	80	160	320



a) Calculate the average rate of change in population for the following intervals of time.

i) $0 \leq t \leq 20$

ii) $20 \leq t \leq 40$

iii) $40 \leq t \leq 60$

iv) $0 \leq t \leq 60$

b) Is the population growth constant?

c) To predict what the population will be in 2050, what assumptions must you make?

9. During the Apollo 14 mission, Alan Shepard hit a golf ball on the Moon. The function $h(t) = 18t - 0.8t^2$ models the height of the golf ball's trajectory on the Moon, where $h(t)$ is the height, in metres, of the ball above the surface of the Moon and t is the time in seconds. Determine the average rate of change of the ball over the time interval $10 \leq t \leq 15$.

10. A company that sells sweatshirts finds that the profit can be modelled by $P(s) = -0.30s^2 + 3.5s + 11.15$, where $P(s)$ is the profit, in thousands of dollars, and s is the number of sweatshirts sold (expressed in thousands).

a) Calculate the average rate of change in profit for the following intervals.

i) $1 \leq s \leq 2$ **ii)** $2 \leq s \leq 3$ **iii)** $3 \leq s \leq 4$ **iv)** $4 \leq s \leq 5$

b) As the number of sweatshirts sold increases, what do you notice about the average rate of change in profit on each sweatshirt? What does this mean?

c) Predict if the rate of change in profit will stay positive. Explain what this means.