Arithmetic Series

Arithmetic Series – the sum of the terms of an arithmetic sequence

examples

1) 1+3+5+7+9+... $a=1$ $d=2$

2) 8+11+14+17+... $a=8$ $d=3$

3) 7+3-1-5... $a=7$ $d=-4$

t_n and S_n formula for each arithmetic sequence examples

General term → $t_n = a(n-1)d$

$S_n = \frac{n}{2}[2a+(n-1)d]$ ← sum of the terms

1) $t_n = 2n-1$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$= \frac{n}{2}[2(1)+(n-1)2]$

$= \frac{n}{2}(2n) = \frac{2n^2}{2} = n^2$

2) $t_n = 3n+5$

$S_n = \frac{n}{2}[2(8)+(n-1)3]$

$= \frac{n}{2}[16+3n-3]$

$= \frac{n}{2}(3n+13)$

$= \frac{3n^2+13n}{2}$

3) $t_n = -4n+11$

$S_n = \frac{n}{2}[2(7)+(n-1)-4]$

$= \frac{n}{2}[14-4n+4]$

$= \frac{n}{2}(-4n+18)$

$= \frac{-4n^2+18n}{2}$

$= -2n^2+9n$

find t_{19} and S_{12} for each arithmetic sequence examples

19th term

Sum of 12 terms

1) $t_{19} = 2(19)-1$

$= 37$

$S_{12} = (12)^2$

$= 144$

2) $t_{19} = 3(19)+5$

$= 57+5$

$= 62$

$S_{12} = \frac{3(12)^2+13(12)}{2}$

$= \frac{432+156}{2}$

$= 294$

3) $t_{19} = -4(19)+11$

$= -65$

$S_{12} = -2(12)^2+9(12)$

$= -288+108$

$= -180$

Ex.1 Find the sum of the following series

(a) 13+18+23+28+...+63 $a=13$ $d=5$

$t_n = 13+(n-1)5$

$= 13+5n-5$

$= 5n+8$

$63 = 5n+8$

$\frac{63-8}{5} = n$

$11 = n$

$S_n = \frac{n}{2}[2(13)+(n-1)5]$

$= \frac{n}{2}(26+5n-5)$

$= \frac{n}{2}(5n+21)$

$= \frac{5n^2+21n}{2}$

$S_{11} = \frac{5(11)^2+21(11)}{2}$

$= \frac{605+231}{2}$

$= 418$

(b) -1+8+17+26+...+143 $a=-1$ $d=9$

$t_n = -1+(n-1)9$

$= -1+9n-9$

$143 = 9n-10$

$\frac{143+10}{9} = n$

$17 = n$

$S_n = \frac{n}{2}[2(-1)+(n-1)9]$

$= \frac{n}{2}[-2+9n-9]$

$= \frac{n}{2}(9n-11)$

$= \frac{9n^2-11n}{2}$

$S_{17} = \frac{9(17)^2-11(17)}{2}$

$= \frac{2601-187}{2}$

$= 1207$