Mistaken Identities

1) $(\sin x + \cos x)(\sin x - \cos x)$ $\qquad 1 - 2\cos^2 x$

$= \sin^2 x - \sin x \cos x + \sin x \cos x - \cos^2 x$

(4) $= \sin^2 x - \cos^2 x$

$= (1 - \cos^2 x) - \cos^2 x$

$= 1 - 2\cos^2 x$

2) $\csc x \, (\sin x - 1)$ $\qquad 1 - \csc x$

$= 1 - \frac{1}{\sin x}$

(4) $= \frac{1}{\sin x}(\sin x - 1)$

$= 1 - \frac{1}{\sin x}$

3) $\csc^2 x - \csc x \cot x$. $\qquad \frac{1}{1 + \cos x}$

(5) $= \csc x \, (\csc x - \cot x)$

$= \frac{1}{\sin x}\left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)$

$= \frac{1}{\sin x}\left(\frac{1 - \cos x}{\sin x}\right)$

$= \frac{1 - \cos x}{\sin^2 x}$

$= \frac{1 - \cos x}{1 - \cos^2 x}$

$= \frac{\cancel{(1 - \cos x)}}{\cancel{(1 - \cos x)}(1 + \cos x)}$

$= \frac{1}{1 + \cos x}$

4) $$\frac{2\sin x\cos x - \cos x}{1 - \sin x + \sin^2 x - \cos^2 x}$$

$\cot x$

$= \dfrac{\cos x}{\sin x}$

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$$= \frac{\cos x\,(2\sin x - 1)}{-\sin x + \sin^2 x + \sin^2 x}$$

$$= \frac{\cos x\,(2\sin x - 1)}{2\sin^2 x - \sin x}$$

$$= \frac{\cos x\,\cancel{(2\sin x - 1)}}{\sin x\,\cancel{(2\sin x - 1)}}$$

$$= \frac{\cos x}{\sin x}$$

5) $(\tan x + \sec x)^2$

$\dfrac{1+\sin x}{1-\sin x}$

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$$= \tan^2 x + 2\tan x\sec x + \sec^2 x$$

$$= \frac{\sin^2 x}{\cos^2 x} + 2\,\frac{\sin x}{\cos^2 x} + \frac{1}{\cos^2 x}$$

$$= \frac{\sin^2 x + 2\sin x + 1}{\cos^2 x}$$

$$= \frac{(\sin x + 1)^2}{(1 - \sin^2 x)}$$

$$= \frac{(\sin x + 1)\cancel{(\sin x + 1)}}{(1 - \sin x)\cancel{(1 + \sin x)}}$$

$$= \frac{1 + \sin x}{1 - \sin x}$$