2. Simplify. Express each answer with positive exponents.

a) $y^{10}(y^4)^{-3}$

b) $(x^{-3})^{-3}(x^{-1})^5$

c) $\dfrac{(n^{-4})^3}{(n^{-3})^{-4}}$

d) $\dfrac{w^4(w^{-3})}{(w^{-2})^{-1}}$

e) $\dfrac{(x^{-1})^4 x}{x^{-3}}$

f) $\dfrac{(b^{-7})^2}{b(b^{-5})b^9}$

3. Consider the expression $\dfrac{x^7(y^2)^3}{x^5y^4}$.

a) Substitute $x = -2$ and $y = 3$ into the expression, and evaluate it.

b) Simplify the expression. Then substitute the values for x and y to evaluate it.

c) Which method seems more efficient?

PRACTISING

4. Simplify. Express answers with positive exponents.

a) $(pq^2)^{-1}(p^3q^3)$

b) $\left(\dfrac{x^3}{y}\right)^{-2}$

c) $\dfrac{(ab)^{-2}}{b^5}$

d) $\dfrac{m^2n^2}{(m^3n^{-2})^2}$

e) $\dfrac{(w^2x)^2}{(x^{-1})^2w^3}$

f) $\left(\dfrac{(ab)^{-1}}{a^2b^{-3}}\right)^{-2}$

5. Simplify. Express answers with positive exponents.

a) $(3xy^4)^2(2x^2y)^3$

b) $\dfrac{(2a^3)^2}{4ab^2}$

c) $\dfrac{(10x)^{-1}y^3}{15x^3y^{-3}}$

d) $\dfrac{(3m^4n^2)^2}{12m^{-2}n^6}$

e) $\dfrac{p^{-5}(r^3)^2}{(p^2r)^2(p^{-1})^2}$

f) $\left(\dfrac{(x^3y)^{-1}(x^4y^3)}{(x^2y^{-3})^{-2}}\right)^{-1}$

6. Simplify. Express answers with positive exponents.

K

a) $(x^4)^{\frac{1}{2}}(x^6)^{-\frac{1}{3}}$

b) $\dfrac{9(c^8)^{0.5}}{(16c^{12})^{0.25}}$

c) $\dfrac{\sqrt{25m^{-12}}}{\sqrt{36m^{10}}}$

d) $\sqrt[3]{\dfrac{(10x^3)^2}{(10x^6)^{-1}}}$

e) $\left(\dfrac{(32x^5)^{-2}}{(x^{-1})^{10}}\right)^{0.2}$

f) $\dfrac{\sqrt[10]{1024x^{20}}}{\sqrt[9]{512x^{27}}}$

7. Evaluate each expression. Express answers in rational form with positive exponents.

a) $(16x^6y^4)^{\frac{1}{2}}$ for $x = 2, y = 1$

b) $\dfrac{(9p^{-2})^{\frac{1}{2}}}{6p^2}$ for $p = 3$

c) $\dfrac{(81x^4y^6)^{\frac{1}{2}}}{8(x^9y^3)^{\frac{1}{3}}}$ for $x = 10, y = 5$

d) $\left(\dfrac{(25a^4)^{-1}}{(7a^{-2}b)^2}\right)^{\frac{1}{2}}$ for $a = 11, b = 10$

8. Evaluate. Express answers in rational form with positive exponents.

   a) $(\sqrt{10\,000x})^{\frac{3}{2}}$ for $x = 16$

   b) $\left(\frac{(4x^3)^4}{(x^3)^6}\right)^{-0.5}$ for $x = 5$

   c) $(-2a^2b)^{-3}\sqrt{25a^4b^6}$ for $a = 1, b = 2$

   d) $\sqrt{\frac{(18m^{-5}n^2)(32m^2n)}{4mn^{-3}}}$ for $m = 10, n = 1$

9. Simplify. Express answers in rational form with positive exponents.

   a) $(36m^4n^6)^{0.5}(81m^{12}n^8)^{0.25}$

   b) $\left(\frac{(6x^3)^2(6y^3)}{(9xy)^6}\right)^{-\frac{1}{3}}$

   c) $\left(\frac{\sqrt{64a^{12}}}{(a^{1.5})^{-6}}\right)^{\frac{2}{3}}$

   d) $\left(\frac{(x^{18})^{\frac{-1}{6}}}{\sqrt[5]{243x^{10}}}\right)^{0.5}$

10. If $M = \frac{(16x^8y^{-4})^{\frac{1}{4}}}{32x^{-2}y^8}$, determine values for x and y so that

    a) $M = 1$ b) $M > 1$ c) $0 < M < 1$ d) $M < 0$

11. The volume and surface area of a cylinder are given, respectively, by the formulas

$$V = \pi r^2 h \quad \text{and} \quad SA = 2\pi rh + 2\pi r^2.$$

    a) Determine an expression, in simplified form, that represents the surface area-to-volume ratio for a cylinder.

    b) Calculate the ratio for a radius of 0.8 cm and a height of 12 cm.

12. If $x = -2$ and $y = 3$, write the three expressions in order from least to greatest.

$$\frac{y^{-4}(x^2)^{-3}y^{-3}}{x^{-5}(y^{-4})^2}, \frac{x^{-3}(y^{-1})^{-2}}{(x^{-5})(y^4)}, (y^{-5})(x^5)^{-2}(y^2)(x^{-3})^{-4}$$

13. How is simplifying algebraic expressions like simplifying numerical ones? How is it different?

Extending

14. a) The formula for the volume of a sphere of radius r is $V(r) = \frac{4}{3}\pi r^3$. Solve this equation for r. Write two versions, one in radical form and one in exponential form.

    b) Determine the radius of a sphere with a volume of $\frac{256\pi}{3}$ m^3.

15. Simplify $\frac{\sqrt{x(x^{2n+1})}}{\sqrt[3]{x^{3n}}}$, $x > 0$