MHF4U: Test – Functions: Understanding Rates of Change
REVIEW

Name: Solutions Mark: KU APP COMM

1. Describe the transformation and state the mapping that will transform $y = f(x)$ to [COMM / 5]

A) $y = -3f(\frac{1}{2}x)$ Description:

[2]

reflection x-axis
Vertical stretch x 3
horizontal stretch x 2

Mapping Notation: $(x,y) \rightarrow$ $(2x, -3y)$

B) $y = -2f(2x-6)$ Description:

[3]

$y = -2f(2(x-3))$

reflection x-axis
vertical stretch x2
horizontal stretch x $\frac{1}{2}$
right 3

Mapping Notation: $(x,y) \rightarrow$ $(\frac{1}{2}x+3, -2y)$

2. Determine the equation of the function if $y = \sqrt{x}$ is transformed to the right 3 units, reflected over the y-axis and vertically stretched by a factor of 2. [APP/2]

$y = 2\sqrt{-(x-3)}$ or $y = 2\sqrt{-x+3}$

3. If $f(x)=\frac{1}{2}(x+3)^2-1$ then,

A) Sketch both $f(x)$ and $f^{-1}(x)$ in the space provided. [APP/4]

$y=x^2$

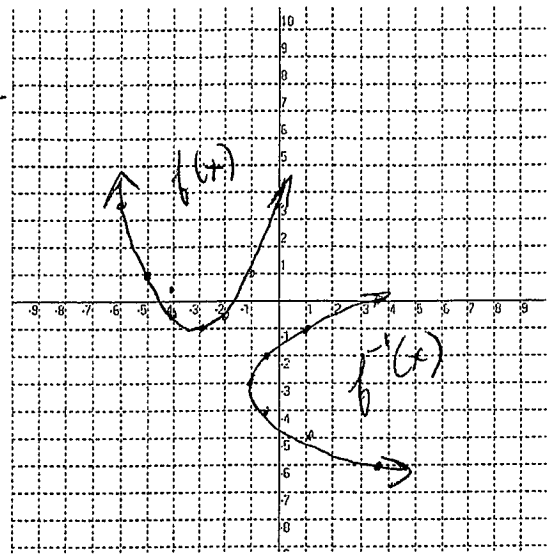
x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$(x,y)\rightarrow$
$(x-3, \frac{1}{2}y-1)$

x	y
-6	3.5
-5	1
-4	-0.5
-3	-1
-2	-0.5
-1	1
0	3.5

INVERSE

x	y
3.5	-6
1	-5
-0.5	-4
-1	-3
-0.5	-2
1	-1
3.5	0



B) From the graph above, determine the domain and range of $f^{-1}(x)$. [KU/1]

$D=\{x \mid x \geq -1, x \in R\}$

$R=\{y \mid y \in R\}$

C) Determine the equation for $f^{-1}(x)$. [APP/3]

$y=\frac{1}{2}(x+3)^2-1$

INV $\quad x=\frac{1}{2}(y+3)^2-1$

$x+1=\frac{1}{2}(y+3)^2$

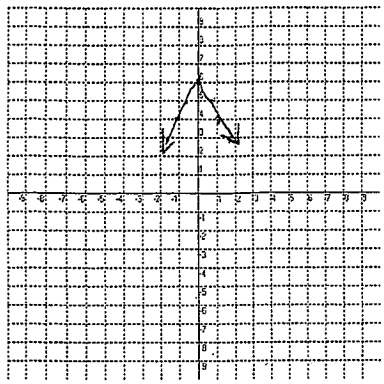
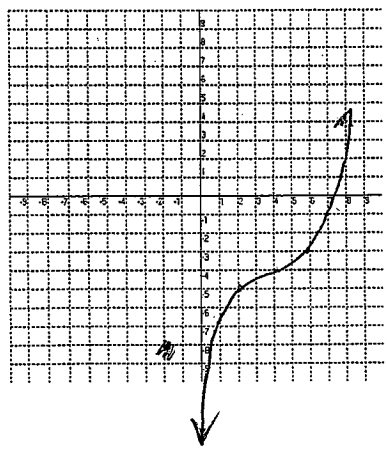
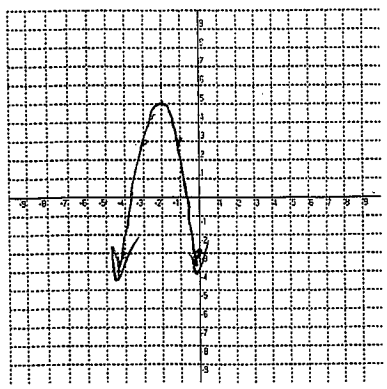
$2x+2=(y+3)^2$

$\pm\sqrt{2x+2}=y+3$

$-3\pm\sqrt{2x+2}=y$

$\therefore f^{-1}(x)=-3\pm\sqrt{2x+2}$

4. Sketch the following functions in the space provided. [KU/10]

Equation	Transformation/Mapping	Graph
$y=-\lvert 2x\rvert+6$	$(x,y)\rightarrow\left(\frac{1}{2}x,\ -y+6\right)$ $\begin{array}{c\|c} x & y \\ \hline -3 & 3 \\ -2 & 2 \\ -1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{array} \rightarrow \begin{array}{c\|c} x & y \\ \hline -1.5 & 3 \\ -1 & 4 \\ -.5 & 5 \\ 0 & 6 \\ .5 & 5 \\ 1 & 4 \\ 1.5 & 3 \end{array}$	
$y=(\frac{1}{2}x-2)^3-4$ $y=\left(\frac{1}{2}(x-4)\right)^3-4$	$(x,y)\rightarrow(2x+4,\ y-4)$ $\begin{array}{c\|c} x & y \\ \hline -2 & -8 \\ -1 & -1 \\ 0 & 0 \\ 1 & 1 \\ 2 & 8 \end{array} \rightarrow \begin{array}{c\|c} x & y \\ \hline 0 & -12 \\ 2 & -5 \\ 4 & -4 \\ 6 & -3 \\ 8 & 4 \end{array}$	
$y=-2x^2-8x-3$ $y=-2(x^2+4x)-3$ $y=-2(x^2+4x+4-4)-3$ $y=-2(x+2)^2+5$ $(x,y)\rightarrow(x-2,\ -2y+5)$ $\begin{array}{c\|c} x & y \\ \hline -3 & 9 \\ -2 & 4 \\ -1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 2 & 4 \\ 3 & 9 \end{array} \rightarrow \begin{array}{c\|c} x & y \\ \hline -5 & -13 \\ -4 & -3 \\ -3 & 3 \\ -2 & 5 \\ -1 & 3 \\ 0 & -3 \\ 1 & -13 \end{array}$	

5. Sketch the piecewise function $f(x)$ on the graph given: [KU/5] $f(x)=\begin{cases}-2x, x<-4\\ 3, -4\le x<2\\ x^3-3, x\ge 2\end{cases}$

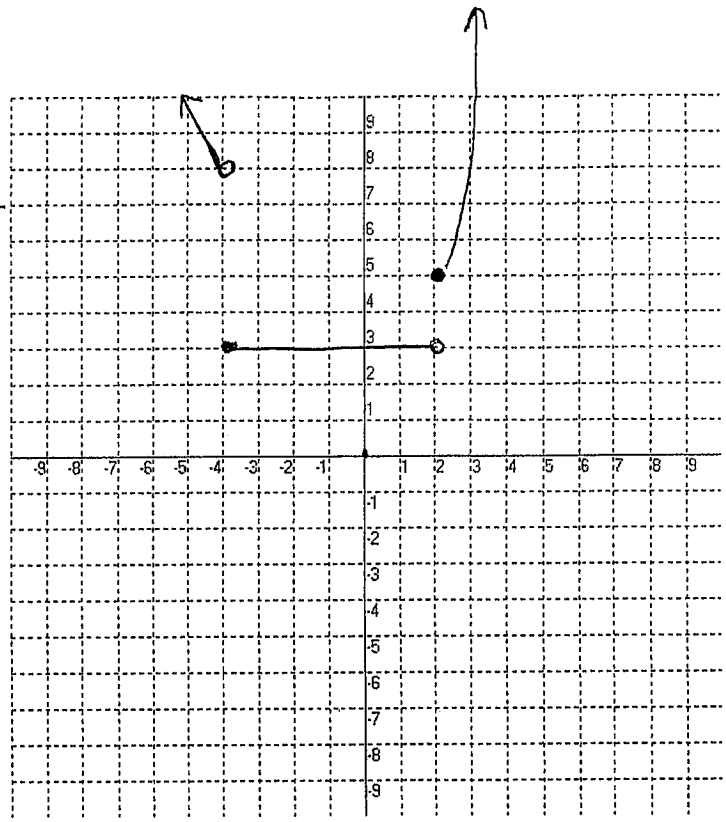
$y=x^3-3$

$(x,y)\to(x,y-3)$

x	Y
-2	-8
-1	-1
0	0
1	1
2	8
3	27

→

x	y
-2	-11
-1	-4
0	-3
1	-2
2	5
3	24



Determine:

$f(-6)=12$

$f(-4)=3$

$f(2)=5$

$f(4)=61$

6. ~~Determine the~~ value of "k" so that the piecewise function $f(x)$ is "continuous" throughout its domain? [APP/2]

$$f(x)=\begin{cases}x^2+4x+k, x<-3\\ \frac{2}{3}x+4, x\ge -3\end{cases}$$

For a continuous fuction, the 2 "pieces" must be equal at $x=-3$

$$x^2+4x+K=\frac{2}{3}x+4$$

@ $x=-3$ $\quad (-3)^2+4(-3)+K=\frac{2}{3}(-3)+4$

$9-12+K=-2+4$

$-3+K=2$

$K=5$

7. Estimate the instantaneous rate of change for the function $f(x) = x^3 + 2x$ at $x = -2$ using the difference-quotient method. Is the function increasing or decreasing when $x = -2$? [KU4]

y	-12	-12.014006
x	-2	-2.001

$$\text{Inst R.O.C} = \frac{-12.014006 - (-12)}{-2.001 - (-2)}$$

$$= 14$$

∴ Since slope of tangent is positive, the function is increasing at $x = -2$

8. An automobile enters a road and travels the following distance in metres during the next 10 seconds, where s represents the distance in metres and t time in seconds.

x	t	0	2	4	6	8	10
y	s	0	7	16	27	40	55

A) Determine the average rate of change (speed) of the vehicle over the first 40 metres? [KU2]

y	0	40
x	0	8

$$\text{avg R.O.C} = \frac{40-0}{8-0}$$

$$= 5 \text{ m/s}$$

B) Estimate the Instantaneous rate of change of the vehicle when $t = 4$ seconds using the "Preceding/Following" Method. [KU4]

Preceding

y	7	16
x	2	4

$$\text{avg R.O.C} = \frac{16-7}{4-2} = \frac{9}{2}$$

Following

y	16	27
x	4	6

$$\text{avg R.O.C} = \frac{27-16}{6-4} = \frac{11}{2}$$

$$\text{Inst R.O.C} = \frac{\frac{9}{2} + \frac{11}{2}}{2}$$

$$= 5 \text{ m/s}$$

@ $t = 4$ sec

C) If the points above satisfy the equation $s = at^2 + bt$, determine a and b then calculate the speed of the car when $t = 18.5$ seconds. (Use any Method) [A6]

① $(2, 7)$

$7 = a(2)^2 + b(2)$

$4a + 2b = 7$

② $(4, 16)$

$16 = a(4)^2 + b(4)$

$16 = 16a + 4b$

$2 \times$ ① $8a + 4b = 14$

② $16a + 4b = 16$

$-8a = -2$

$a = \frac{1}{4}$ $\quad b = 3$

$s = \frac{1}{4}t^2 + 3t$

y	141.0625	141.063725
x	18.5	18.5001

Inst R.O.C = 12.25 m/s