Trig Applications Review

#1 The rodent population varies with the number of predators that inhabit the region. At any time you can predict the rodent population (r) using the function $r = 2500 + 1500\sin\left(\frac{\pi}{4}t\right)$ where t is the number of years that have passed since 1976.

(a) Graph the function for 3 cycles.
(b) What is the period of the function?
(c) In the first cycle of the function, what was the maximum number of rodents and in which year did it occur?
(d) What was the minimum number of rodents and in which year did it occur?
(e) How many rodents would you predict for the year 2014?
(f) Determine the average rate of change of the rodent population from the year 2000 to 2007.
(g) Determine the instantaneous rate of change of the rodent population in the year 2008.

#2 A group of students is tracking a friend, John, who is riding a Ferris Wheel. They start tracking his height and see that John reaches the maximum height of 40 m after 3 sec and reaches a minimum height of 5 m after 25 seconds.

(a) Graph the function for 3 cycles.
(b) Determine a sinusoidal function that models John's height with respect to time. Use two different equations, one using sine and another using cosine.
(c) Determine John's height 3 minutes after the students start tracking his height.
(d) Determine John's initial height at the time when the friends start tracking.

#3 A group of students is tracking a friend, John, who is riding a Ferris Wheel. John gets on at the bottom, which is 4 m off the ground, and they watch John reach a maximum height of 36 m after 12 seconds. John continues to enjoy the ride, and after 5 times around, he gets off at the bottom.

(a) Graph the function for 3 cycles.
(b) Determine a sinusoidal function that models John's height with respect to time. Use two different equations, one using sine and another using cosine.
(c) Determine John's height after 1 minute and 10 seconds.
(d) John is able to see the falls when he is above 32 m. For each cycle, how long is the falls visible to John?

Example 1

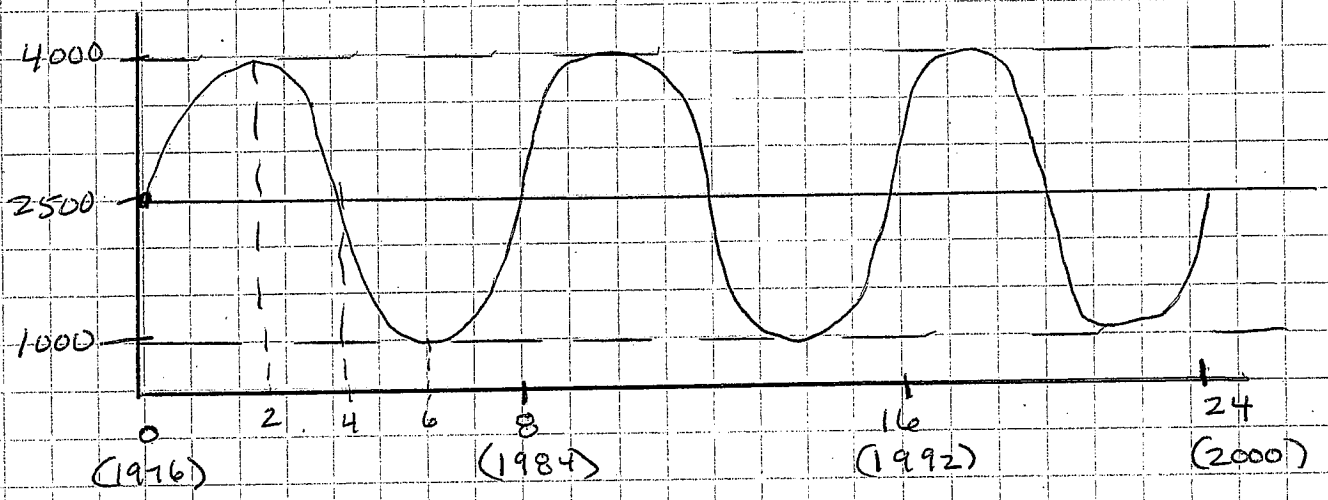
1. $y = 1500 \sin\left(\frac{\pi}{4}t\right) + 2500$

amplitude

$PL = \frac{2\pi}{\pi/4} = 8$

no phase shift

↑ central axis $y = 2500$



a) graph

b) PL = 8 years

c) max = 4000 in 1978

d) min = 1000 in 1982

e) 2014 is 38 years after 1976

so sub in $t = 38$

$y = 1500 \sin(\pi/4(38)) + 2500$

$= 1000$

∴ 1000 rodents

f) avg ROC

y	2500	1439.3
x	24	31

$$\text{avg ROC} = \frac{1439.3 - 2500}{31 - 24} = -151.53$$

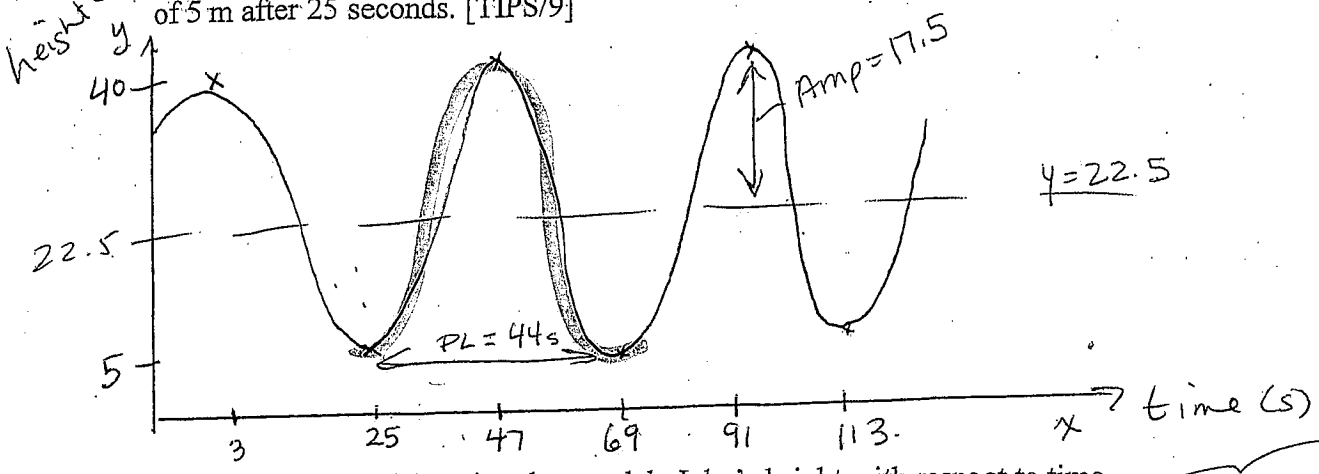
g) Inst ROC

y	2500	2501.178097
x	32	32.001

INST ROC = 178.1

EXAMPLE 2

10. A group of students is tracking a friend, John, who is riding a Ferris wheel. They know that John reaches a maximum height of 40 m after 3 seconds and then reaches the minimum height of 5 m after 25 seconds. [TIPS/9]



A) Determine a sinusoidal function that models John's height with respect to time.

$$y = -17.5\cos\left(\frac{\pi}{22}(x-25)\right)+22.5$$

Period

$PL = \frac{2\pi}{k}$

$k = \frac{2\pi}{44}$

$k = \frac{\pi}{22}$

B) Determine John's height 3 minutes into the ride? Is he rising or falling?

$x = 180$

$$y = -17.5\cos\left(\frac{\pi}{22}(180-25)\right)+22.5$$
$$= -17.5(-0.98982)+22.5$$
$$= 17.32 + 22.5$$
$$= 39.82$$

∴ height is 39.82 m.

Falling

C) Determine John's initial height?

$x = 0$

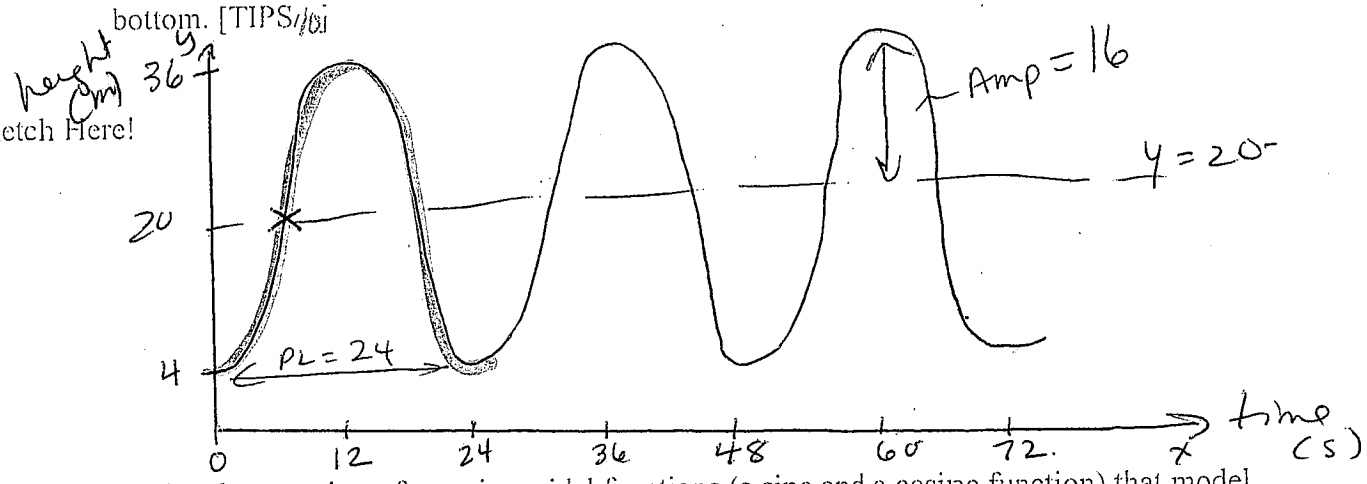
$$y = -17.5\cos\left(\frac{\pi}{22}(0-25)\right)+22.5$$
$$= -17.5(-0.90963)+22.5$$
$$= 15.92 + 22.5$$
$$= 38.42$$

∴ initial height is 38.42 m.

EXAMPLE 3

A group of students is tracking a friend, John, who is riding a Ferris wheel. John gets on at the bottom, which is 4 m off the ground, and they watch John reach a maximum height of 36 m after 12 seconds. John continues to enjoy the ride, and after 5 times around, he gets off at the bottom. [TIPS/6]

Sketch Here!



A) Determine the equation of two sinusoidal functions (a sine and a cosine function) that model John's height with respect to time.

$$y = -16\cos\left(\frac{\pi}{12}(x)\right) + 20$$

$$y = 16\sin\left(\frac{\pi}{12}(x-6)\right) + 20$$

Period

$PL = \frac{2\pi}{k}$

$k = \frac{2\pi}{24}$

$k = \frac{\pi}{12}$

B) How high is John after 1 min and 10 seconds?

$x = 70$

$$y = -16\cos\left(\frac{\pi}{12}(70)\right) + 20$$
$$= -16(0.866025) + 20$$
$$= 6.14\text{ m}$$

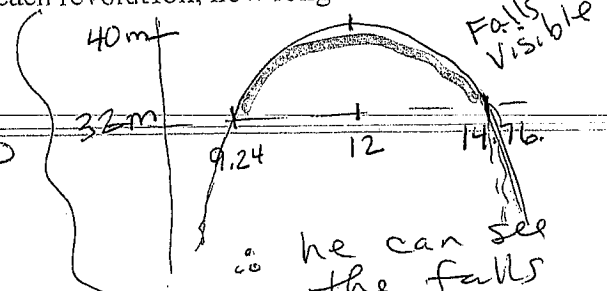
∴ John is 6.14 m high

C) John is able to see the Falls when he is above 32 m. For each revolution, how long is the Falls visible?

$h = 32$

$$32 = -16\cos\left(\frac{\pi}{12}(x)\right) + 20$$
$$12 = -16\cos\left(\frac{\pi}{12}(x)\right)$$
$$-0.75 = \cos\left(\frac{\pi}{12}(x)\right)$$
$$138.59 = \frac{\pi}{12}(x)$$
$$9.24 \doteq x$$

↖ he reaches a height of 32 m after 9.24 seconds.



∴ he can see the falls for 5.52 sec.